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FORMATION AND EVOLUTION OF A 0.242 $M_{\odot}$ HELIUM WHITE DWARF IN THE PRESENCE OF ELEMENT DIFFUSION

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ABSTRACT

The evolution of a 0.242 $M_{\odot}$ object that finally becomes a helium white dwarf is modeled from Roche lobe detachment down to very low luminosities ($\log L/L_{\odot} = -5$). In doing so, we employ our stellar code, to which we have added a set of routines that compute the effects due to gravitational settling and chemical and thermal diffusion. Initial models are constructed by abstracting mass from a 1 $M_{\odot}$ red giant branch model up to the moment at which the model begins to evolve blueward. From then on, two detailed sequences have been computed: one sequence with element diffusion and the other without that phenomenon. Results without diffusion are very similar to those of Driebe and collaborators. We find that element diffusion introduces important changes in the internal structure of the star. In particular, models with diffusion undergo *three* thermonuclear flashes, whereas models without diffusion experience only one. This fact has a large effect on the fraction of total hydrogen mass left in the star (about 3 times less hydrogen compared to models without diffusion) at the start of the final cooling track. As a result, at late stages of evolution models with diffusion are characterized by a much smaller nuclear energy release. Consequently, the star has to take energy from its relic thermal content, causing its further evolution to be significantly faster compared with the standard treatment. Notably, these new, more detailed structures strongly resemble those we have assumed in previous work on helium white dwarfs with hydrogen envelopes.

Conventional wisdom indicates that a millisecond pulsar is recycled during the mass transfer stage in a binary system. Usually, the companion to the pulsar is a low-mass white dwarf. If zero ages are set at the end of mass transfer, the ages of both objects should be the same. Available models characterized by dominant hydrogen burning lead to a strong discrepancy between the ages of PSR B1855+09 and its white dwarf companion. We interpret such a discrepancy as a direct consequence of ignoring element diffusion in the stellar models. We show that in the frame of models in which diffusion is properly accounted for, ages naturally come into a nice agreement. Consequently, we do not have to invoke any ad hoc mass loss or exotic mechanisms to account for the ages of the stars that belong to the binary system PSR B1855+09.

Subject headings: pulsars: general — stars: evolution — stars: interiors — white dwarfs

1. INTRODUCTION

In recent years, some low-mass white dwarf (WD) stars have been detected as companions to millisecond pulsars. Such systems are very interesting in the sense that they allow us to infer characteristics of one component of the system from studying the physical characteristics of the other. Usually, the WDs detected have a very low mass, lower than the threshold for the ignition of helium-burning reactions ($\approx 0.45\ M_{\odot}$). For this reason, such objects are suspected to be WDs with helium-rich interiors, i.e., helium WDs.

Helium WDs have captured the attention of many researchers, who have devoted much effort to their study. Nowadays, it is well established that helium WDs should have been formed in a binary system, because an isolated star would require a timescale much longer than the present age of our universe to reach such a configuration. Recent studies of helium WD evolution include Althaus & Benvenuto (1997), Benvenuto & Althaus (1998), Hansen & Phinney (1998), Driebe et al. (1998), Sarna, Antipova, & Ergma (1999), and Althaus & Benvenuto (2000). Other previous studies devoted to studying the evolution of these objects are Kippenhahn, Kohl, & Weigert (1967), Kippenhahn, Thomas, & Weigert (1968), Webbink (1975), and Iben & Tutukov (1986).

A crucial issue in the study of helium WDs is the amount of hydrogen left atop the WD after mass transfer episodes. Calculations performed by the Estonian group (Sarna et al. 1999) are particularly noteworthy in this regard. Indeed, Sarna et al. have presented very detailed calculations of the pre-WD evolution during the mass transfer stage in a close binary system. They found that, after detachment of the Roche lobe, low-mass helium WDs are surrounded by massive hydrogen envelopes of 0.01–0.06 $M_{\odot}$, with a surface hydrogen abundance by mass of $X_{\rm H} = 0.35$–0.5. Massive hydrogen envelopes have also been derived by the Potsdam group (Driebe et al. 1998), who mimicked the binary evolution by abstracting mass from a 1 $M_{\odot}$ model at an adequate rate.

Binary systems composed of a helium WD and a millisecond pulsar offer a good opportunity to test the predictions of evolutionary calculations. As a matter of fact, millisecond pulsars are thought to be recycled during the mass transfer stage (Bhattacharya & van den Heuvel 1991).

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When mass transfer ends, the pulsar begins to spin down. If we set zero ages at the end of such a mass transfer, then the ages of the WD and the pulsar component should be the same. The age of a pulsar can be estimated employing its current period P and its rate of variation $\dot{P}$. If we assume $\dot{P} \propto P^n$ (where n is the so-called braking index), then the age of the pulsar t_{PSR} is

$$t_{PSR} = [P/(n-1)\dot{P}][1 - (P_0/P)^{n-1}] , \quad (1)$$

where P_0 is the spin period at the end of mass transfer. If $P_0 \ll P$ and $n = 3$ (which corresponds to the case of dipolar radiation), then

$$t_{PSR} = P/2\dot{P} . \quad (2)$$

A well-studied system of this type is PSR J1012+5307, for which the age of the WD component, computed with models in which nuclear burning is dominant (Driebe et al. 1998; Sarna et al. 1999), is in good agreement with the spin-down age of the pulsar. However, this is not the case for the system PSR B1855+09. In fact, very recently van Kerkwijk et al. (2000) have optically detected the low-mass component of this system and determined its effective temperature to be 4800 ± 800 K. In addition, the mass of the WD is accurately known, thanks to the measure of the Shapiro delay of the pulsar signal, to be $0.258^{+0.028}_{-0.016}$ $M_\odot$. Use of the Driebe et al. (1998) models results in an age for the WD component twice as long as the pulsar spin-down age (≈5 Gyr; see van Kerkwijk et al. 2000). This may be considered a problem with the evolutionary models for such WDs. Notably, the models that have led to such an unexpected result have neglected diffusion.

In the context of WD evolution, element diffusion has been considered by Iben & MacDonald (1985) for the case of a carbon-oxygen 0.6 $M_\odot$ object. Furthermore, Iben & Tutukov (1986) computed the evolution of a 0.296 $M_\odot$ helium WD. In our opinion, Iben & Tutukov (1986) is a very important paper the results of which have been somewhat overlooked. They modeled a 0.296 $M_\odot$ object through all its evolutionary stages and found two thermonuclear flashes; these results resemble the ones we shall present below. Remarkably, they found that after flashes the amount of hydrogen in the envelope of the model is unable to support an appreciable amount of hydrogen burning, forcing a fast evolution for the resulting WD star. Interestingly enough, they did not compute a sequence without diffusion, so it was not clear if diffusion was the main agent in inducing thermonuclear flashes in their models.

It is the purpose of this work to study the effect of element diffusion on the structure and evolution of a 0.242 $M_\odot$ helium WD star. This mass value should be representative of the companion to PSR B1855+09. In doing so, we shall employ a detailed stellar code with which two sequences will be computed: one with and the other without diffusion. Thus, the resulting differences should be due to the effects of that process.

The rest of this paper is organized as follows. In § 2 we briefly describe the main characteristics of the computer code we have employed. In § 3 we describe our numerical results. Finally, we discuss the implications of our findings in § 4.

2. THE EVOLUTIONARY CODE

For the present work, we shall employ the same stellar code we used in our previous works on WD evolution (Althaus & Benvenuto 1997, 2000; Benvenuto & Althaus 1998). The code is based on a very up-to-date and detailed physical description. In particular, we consider OPAL radiative opacities (Iglesias & Rogers 1996) for different metallicities, consistent with the expectations from element diffusion, and a complete network for hydrogen-burning reactions. The equation of state for the low-density regime is an updated version of that of Magni & Mazzitelli (1979), while for the high-density regime we consider ionic contribution, Coulomb interactions, partially degenerate electrons, and electron exchange and Thomas-Fermi contributions at finite temperature. In addition, important modifications have been incorporated, allowing us to compute some extremely fast evolutionary stages that are critical for the problem at hand. Essentially, we have transformed our code to apply the Henyey scheme to the differences in the physical quantities (luminosity, pressure, radius, and temperature) between the previous and the computed model, as recommended by Ziółkowski (1970).

For the computations presented in this paper, we have included gravitational settling and chemical and thermal diffusion. Briefly, diffusion calculations are based on the multicomponent treatment developed by Burgers (1969). In order to calculate self-consistently the dependence of the evolution of our WD models on the varying abundance, diffusion calculations have been coupled to our evolutionary code. We refer the reader to Althaus & Benvenuto (2000) for further details about our treatment of diffusion. Both evolutionary sequences (with and without diffusion) we present here comprise about 30,000 models, each divided into about 2000 mesh points.

Because some stages of evolution we shall present below are very fast, it should be stressed that our code is a hydrostatic equilibrium one and also that convective currents are assumed to mix convection zones instantaneously. With regard to the outer boundary condition we followed the standard treatment of a gray atmosphere. For the effect of such approximations on the final results, see § 4.

3. EVOLUTIONARY RESULTS

In order to have a realistic starting model, we have employed the technique of abstracting mass from a 1 $M_\odot$ giant star up to the moment it begins to evolve to the blue part of the H-R diagram. In this way we have simulated the mass transfer episode during Roche lobe overflow. From then on, the evolution of the model has been followed, assuming a constant value for the stellar mass down to a luminosity of $\log L/L_\odot = -5$. It is important to remark that the results we have calculated without diffusion are in very good agreement with those presented by Driebe et al. (1998); thus, in comparing our results with and without diffusion we are also comparing them with the results of the Potsdam group.

The evolutionary tracks for the models with and without diffusion are shown in Figure 1. Labels along the track from A to Q represent evolutionary stages for models with diffusion, the main characteristics of which are listed in Table 1. While the standard treatment predicts the occurrence of one loop, models with diffusion undergo three loops. Each of these loops is due to unstable nuclear burning at the bottom of the hydrogen-rich envelope, i.e., thermonuclear flashes. Because these loops are in fact very short evolutionary stages, there is very little chance to observe stars under such conditions. However, these flashes are critical events that, to

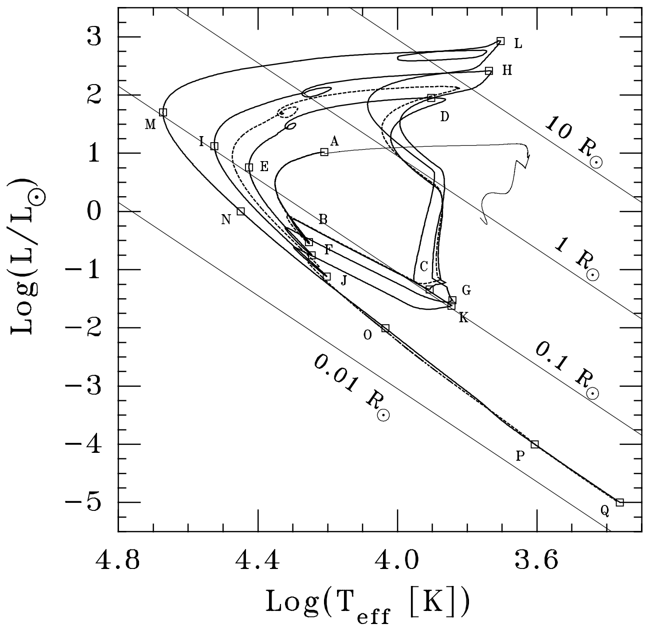

FIG. 1.—Evolutionary tracks for a 0.242 $M_\odot$ object that finally reaches a helium WD structure. *Thick solid line*: models with element diffusion; *dashed line*: standard models without diffusion; *thin solid line*: evolution of the 1 $M_\odot$ model from which we abstract mass until it begins to evolve blueward. Selected stages in models with diffusion are labeled with capital letters, and their evolutionary characteristics are listed in Table 1. Note that models with diffusion undergo three loop excursions to the red giant part of the H-R diagram, whereas models without diffusion have only one such episode. These loops are the outer, observable signal that the star has undergone a hydrogen thermonuclear flash. Note, however, that most of these loops are in fact very short evolutionary stages, so there is almost no chance to observe the object under such conditions.

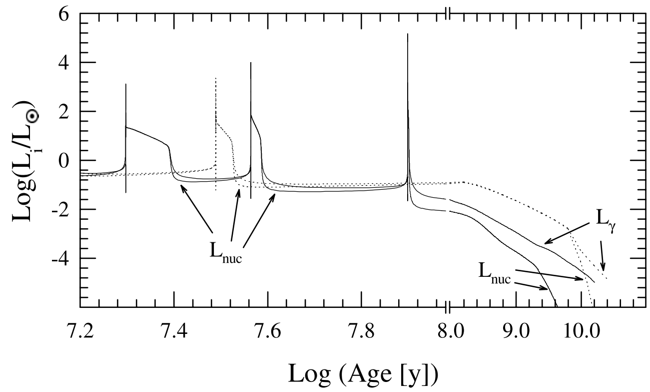

FIG. 2.—Photon and nuclear luminosity as a function of time for the 0.242 $M_\odot$ helium WD models with (*solid lines*) and without (*dashed lines*) element diffusion. Thermonuclear flashes (three in models with diffusion and one in standard models without diffusion) correspond to sharp spikes both in photon and nuclear luminosities. After the spikes there are somewhat long time periods for which both luminosities are high. During such periods, the total amount of hydrogen in the star is considerably reduced.

a large extent, determine the subsequent evolution of the object, even during the final WD cooling phase. The occurrence of three flashes in models with diffusion is in sharp contrast with the predictions of the standard treatment and clearly indicates that element diffusion is by no means a negligible phenomenon in the context of such stars. This is clearly illustrated by Figure 2, where we show the photon and nuclear luminosities as a function of time. In this figure, thermonuclear flashes correspond to sharp spikes in both photon and nuclear luminosities. Note that after spikes there are somewhat long time periods for which both luminosities are high. These periods correspond to evolutionary stages during which the total amount of hydrogen in the star (M_H/M_*) is noticeably reduced, as is apparent from Figure 3. An appreciable reduction of the hydrogen envelope is also found after the only flash in the sequence of models without diffusion. But the key difference between the two sets of models is related to the occurrence of nuclear burning during the final WD cooling phase. In models with diffusion, the fraction of total hydrogen mass left after flash episodes is $M_H/M_* \approx 1.8 \times 10^{-3}$. In the case of models without diffusion, however, at the start of the cooling track the amount of hydrogen that remains is about 3 times as large ($M_H/M_* \approx 5.6 \times 10^{-3}$). As a direct consequence of such a large difference, models without diffusion burn $\frac{2}{3}$ of their hydrogen content *during the cooling track.*

TABLE 1

SELECTED EVOLUTIONARY STAGES OF A 0.242 $M_\odot$ MODEL WITH DIFFUSION

Stage	log L ($L_\odot$)	log T_{eff} (K)	log T_c (K)	log ρ_c (g cm^{-3})	Age (Myr)	log g (cm s^{-2})	log L_{nuc} ($L_\odot$)	X	log M_H (M_*)
A	1.0199	4.2099	7.4791	5.3039	0.4592	4.5962	1.0193	0.6936	−1.892
B	−0.5288	4.2539	7.4849	5.3227	16.3286	6.3208	−0.6185	0.9999	−2.034
C	−1.3393	3.9080	7.4804	5.3184	19.8050	5.7476	3.0263	0.9694	−2.041
D	1.9518	3.9033	7.4801	5.3179	19.8062	2.4378	0.9638	0.5652	−2.042
E	0.7545	4.4258	7.4783	5.3195	23.6620	5.7252	0.7469	0.9995	−2.155
F	−0.7527	4.2454	7.4809	5.3312	29.2864	6.5107	−0.8618	0.9997	−2.167
G	−1.5261	3.8426	7.4717	5.3276	36.6295	5.6728	3.6901	0.9741	−2.177
H	2.4167	3.7377	7.4715	5.3272	36.6300	1.3106	1.6926	0.4112	−2.178
I	1.1232	4.5248	7.4691	5.3259	38.0797	5.7524	1.1148	0.9743	−2.342
J	−1.1185	4.2024	7.4611	5.3501	58.8724	6.7046	−1.2510	0.9997	−2.364
K	−1.6205	3.8451	7.4336	5.3504	79.2183	5.7773	4.9149	0.9998	−2.451
L	2.9307	3.7048	7.4327	5.3490	79.2184	0.6647	2.6762	0.2150	−2.457
M......	1.7033	4.6723	7.4307	5.3465	79.4864	5.7620	1.6941	0.4798	−2.676
N	0.0033	4.4495	7.4343	5.3530	79.8159	6.5709	−0.6076	0.9604	−2.724
O	−2.0065	4.0346	7.2939	5.4108	243.329	6.9214	−2.4387	0.9997	−2.734
P	−4.0018	3.6064	6.4323	5.4945	5343.85	7.2040	−6.4827	0.9998	−2.743
Q	−4.9994	3.3627	5.8342	5.5038	16779.3	7.2266	...	0.9998	−2.743

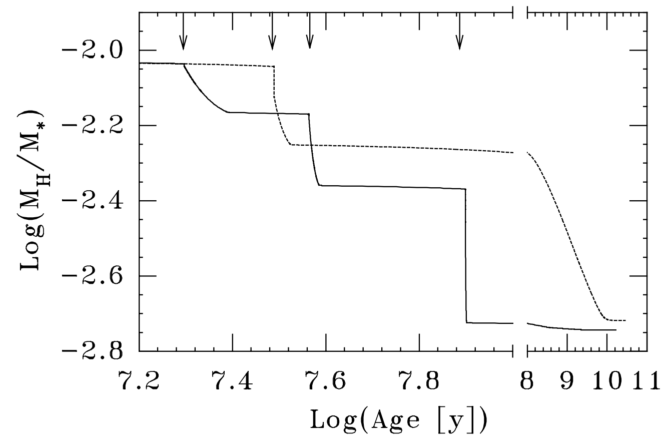

FIG. 3.—Total mass fraction of hydrogen as a function of time for the $0.242\ M_\odot$ helium WD model with (*solid line*) and without (*dashed line*) element diffusion. Arrows indicate the occurrence of thermonuclear flashes. Noticeable hydrogen depletion is found immediately after flashes. Note that models without diffusion enter the final cooling branch ($t > 1$ Gyr) with a much higher hydrogen mass, which is burnt during this epoch. In models with diffusion, nuclear reactions also occur, but at a much lower total rate. This fact has a large effect on the final age of the star.

Nuclear reactions also occur in models with diffusion, but at a much lower total rate. As we shall show below, this fact has a large effect on the final age of the star.

In Figure 4 we show the abundance of hydrogen at $1 - M_r/M_* = 10^{-9}$ as a function of time. As can be expected, the inclusion of diffusion has a large effect on the hydrogen surface abundance and on the star's general outer structure as well. During the evolution after mass transfer, diffusion causes hydrogen to float to the surface. Shortly after a thermonuclear flash begins, the star suddenly increases its radius and develops an outer convection zone (OCZ). Such an OCZ becomes deep enough to reach helium-rich layers, with the result that hydrogen and helium are instantaneously mixed. When the star returns to the blue region of the H-R diagram, the evolutionary timescale gets longer and surface gravity larger, and diffusion begins to be important again. Then, on a timescale much shorter than the evolutionary one, the stellar surface again becomes made only of hydrogen. This is essentially the behavior of the hydrogen and helium abundances when the star goes across the flashing epoch. This is in sharp contrast with the results from models without diffusion. For those models there is no physical agent, other than convection, capable of driving hydrogen to the surface. Thus, final hydrogen abundance is fixed during the last epoch at which the object develops an OCZ.

In order to perform a more detailed exploration of the results of models with diffusion, we shall analyze the evolution of the star during the second loop. In fact, we selected those evolutionary stages of the model from shortly before the ignition of the second hydrogen flash until near the beginning of the third (i.e., last) flash. In Figure 5 we show this portion of the evolutionary track. Along the track we have denoted times before and after the total nuclear energy release per unit time ($L_{\rm nuc}$) reaches its maximum value. We also indicate (with the number of the model in the computed sequence) the location of some models to be employed later in this analysis. Model 9200, for instance, is about a thousand years before maximum $L_{\rm nuc}$. After model 9200, evolution is strongly accelerated, and most of the track toward lower luminosities is crossed in only a hundred years. Shortly after maximum $L_{\rm nuc}$ is reached, convective mixing occurs, and the object evolves very rapidly to high luminosities in a few hundred years. Then it begins to evolve blueward very rapidly, to log $T_{\rm eff} \approx 4.2$. Afterward, from model 22,000 on evolution becomes much slower, slow enough for diffusion to appreciably modify the profile of

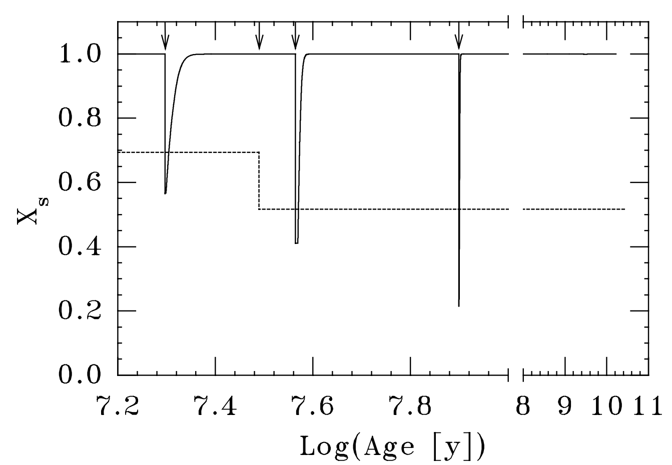

FIG. 4.—Surface abundance of hydrogen as a function of time for the $0.242\ M_\odot$ helium WD model with (*solid line*) and without (*dashed line*) element diffusion. Arrows indicate the occurrence of thermonuclear flashes. After mass transfer, diffusion makes hydrogen float to the surface. When a thermonuclear flash begins, the star develops a very deep outer convection zone that eventually will be able to mix hydrogen and helium. After the luminosity peak, evolution slows down and diffusion takes over again. As a result, hydrogen abundance rapidly goes up. In models without diffusion, however, the final hydrogen abundance is fixed during the occurrence of the last outer convection zone.

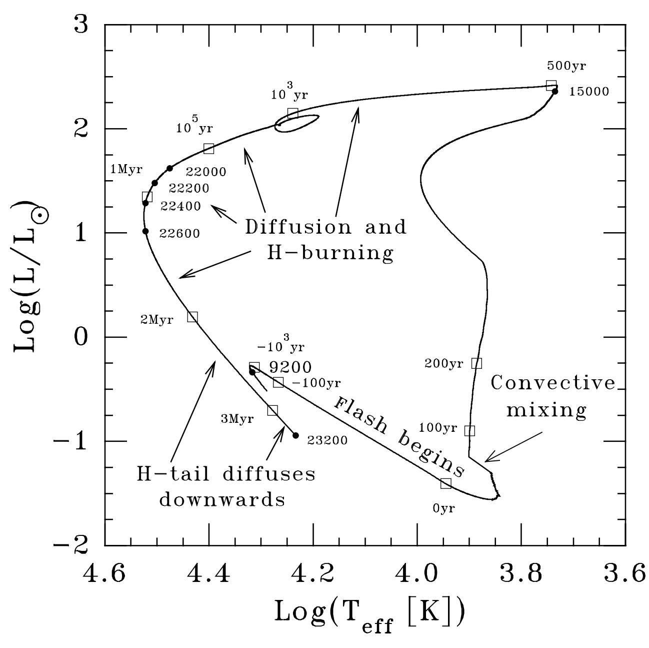

FIG. 5.—Same as Fig. 1, but for the loop corresponding to the second thermonuclear flash experienced by the sequence of models with diffusion. Times indicated by squares along the track correspond to times before and after the maximum nuclear energy release $L_{\rm nuc}$. Some models to be analyzed later are denoted with circles and labeled with their corresponding numbers in the computed sequence. The zero-age point corresponds to the moment at which $L_{\rm nuc}$ reaches its maximum value.

hydrogen abundance in the outer layers of the star. About 3 Myr after the maximum of L_{nuc}, the tail of the hydrogen profile chemically diffuses downward, seeding the occurrence of the third (last) thermonuclear flash.

In Figure 6 we show nuclear and photon luminosities corresponding to the first (fastest) portion of the second thermonuclear flash stage. Here, τ_2 is the age of the model measured from the moment the model reaches its maximum L_{nuc}. As stated above, maximum L_{nuc} occurs about 500 yr before the star reaches its maximum luminosity. For more details about the behavior of the object during these phases, we show in Figure 7 the evolution of log L, log L_{nuc}, log M_H/M_*, and X_S as a function of τ_2 for models belonging to the second loop in the H-R diagram.

Let us analyze the profile of the hydrogen abundance during the second thermonuclear flash. The effects of diffusion are twofold. At the surface, gravitational settling tends to increase hydrogen abundance, because hydrogen is the lightest element present. At the bottom, by contrast, there is a large abundance gradient. Chemical diffusion acts against such a gradient, making hydrogen sink and reach hot enough layers for the ignition of thermonuclear reactions. These, in turn, favor the occurrence of a flash. This is not the only mechanism for inducing a flash in a helium WD star. In fact, models without diffusion also undergo a flash, but after that event there is no mechanism capable of fueling hydrogen to hot layers. This is a key difference between the two sequences of models.

In Figure 8 (*left panel*) we show the hydrogen profile for models between the onset of the flash (model 8800) and the time when diffusion becomes relevant again (model 22,000). Before the flash, hydrogen diffuses outward until the moment the model develops a deep OCZ, forcing hydrogen and helium to mix. As the model returns to the blue region of the H-R diagram, the evolutionary timescale gets longer and therefore diffusion becomes relevant again. This is shown in Figure 8 (*right panel*), where the effect of diffusion on the hydrogen abundance profile for the subsequent models is clearly noticeable. These results indicate the importance of diffusion for adequately computing the composition of such layers, which are responsible for many of the observable characteristics of the object.

In Figure 9 (*left panel*) we show the details of the hydrogen profile at the bottom of the envelope for the same

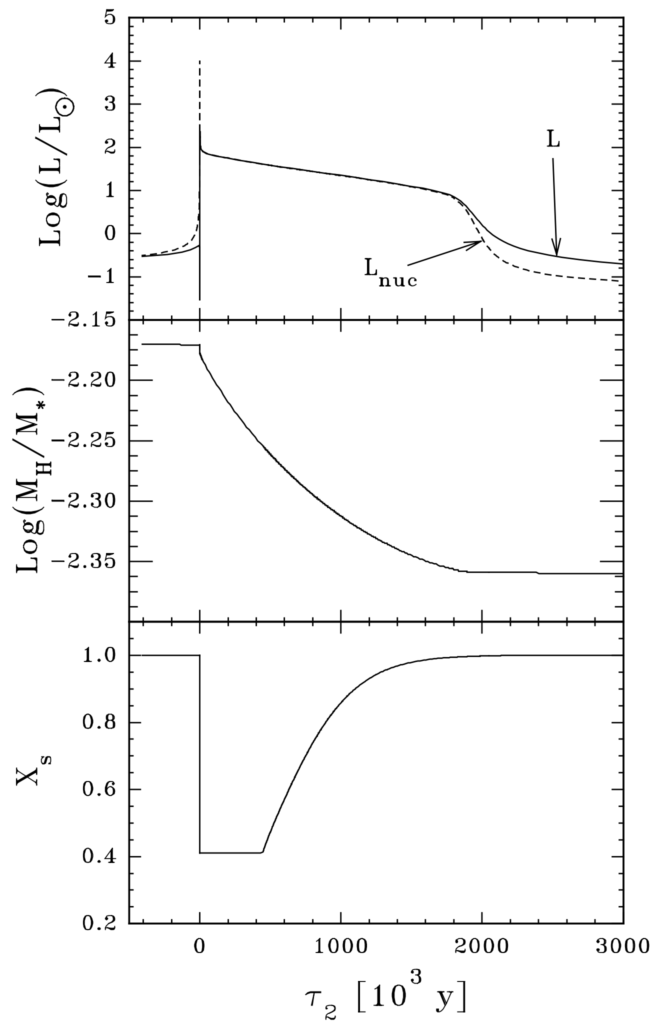

FIG. 7.—Some relevant features of the 0.242 $M_\odot$ model in terms of age for models with diffusion corresponding to the second loop in the HRD. *Top*: Evolution of photon and nuclear luminosity. The spike is due to the occurrence of the thermonuclear flash. *Middle*: Evolution of the hydrogen envelope mass; *bottom*: abundance by mass fraction of hydrogen at $1 - M_r/M_* = 10^{-9}$. Shortly after flash, models develop an OCZ, which leads to hydrogen and helium mixing, markedly reducing the surface abundance of hydrogen. However, diffusion rapidly causes the bulk of hydrogen to float again.

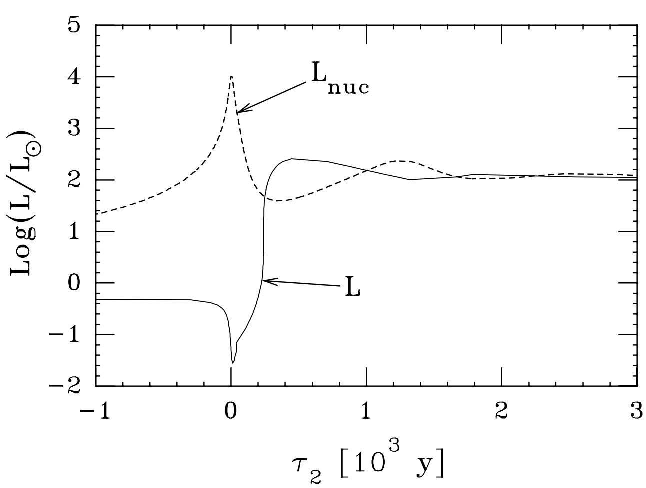

FIG. 6.—Photon and nuclear luminosity as a function of time for the 0.242 $M_\odot$ helium WD model with diffusion. The evolutionary stages depicted correspond to a short time interval during which the second thermonuclear flash happens. Zero-age point is fixed at the maximum nuclear luminosity. Note that at maximum nuclear energy release, the star's luminosity is minimum. However, this trend is reversed in $\approx$400 yr.

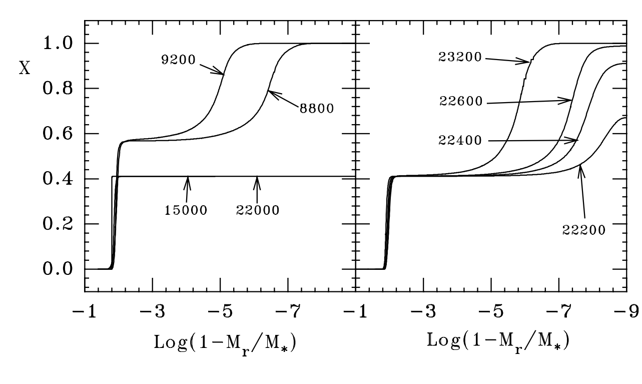

FIG. 8.—Hydrogen profile at selected evolutionary stages as indicated by the model number (see Fig. 5)

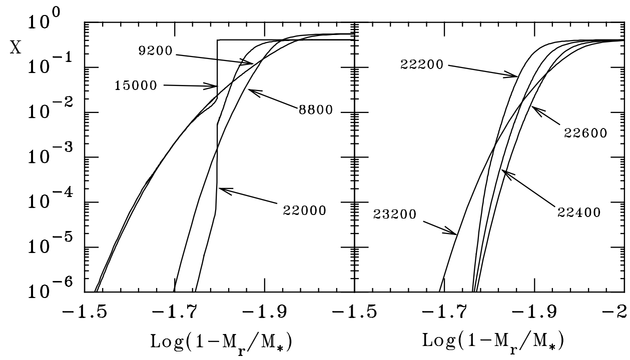

FIG. 9.—Same as Fig. 8, but for the tail of hydrogen distribution

models as in Figure 8. From model 8800 to model 9200 chemical diffusion carries some hydrogen downward deep enough for the star to ignite hydrogen there. Afterward, the object develops an OCZ, which mixes hydrogen and helium, producing the steplike profile of model 15,000. The profile of model 22,000 clearly shows the effects of the subsequent nuclear burning, which makes the profile propagate outward. Later models (Fig. 9, *right panel*) show that, as consequence of nuclear burning, the hydrogen profile continues propagating outward until hydrogen burning sharply decreases (see Fig. 10). After this epoch, chemical diffusion dominates again, carrying some fresh hydrogen to deeper layers. This, in turn, seeds these layers for the last thermonuclear flash.

Now let us discuss the final stages of the evolution of this object. They are relevant in the context of observed helium WDs because they represent the longest period in the life of the 0.242 $M_\odot$ object. From the results presented in Figure 11 (*top panel*), it is clear that models with diffusion undergo a much faster cooling compared with models without diffusion. This is easy to understand in terms of the fraction of nuclear luminosity released by the object (Fig. 11, *bottom panel*). It can be noted that for models without diffusion, nuclear reactions account for almost all the energy radiated by the star. This strongly delays cooling to very long ages (see Driebe et al. 1998 for similar results). In sharp contrast, models with diffusion enter the cooling branch with about one-third the total hydrogen mass of models without diffusion. As a result, the still-occurring nuclear reactions are no longer able to dominate the energetics of the star. Thus, in order to radiate the star must extract energy from the relic thermal content, forcing a much faster cooling. To place this

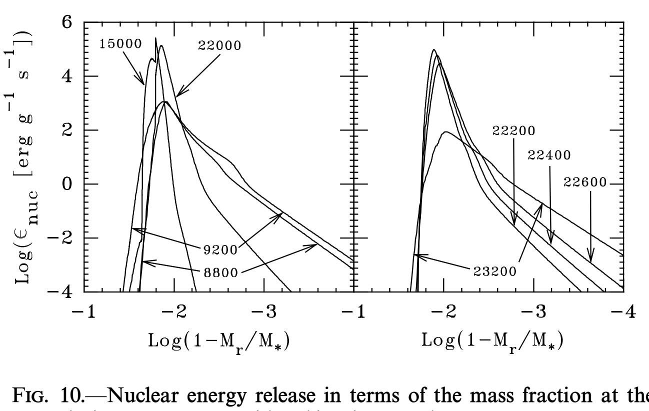

FIG. 10.—Nuclear energy release in terms of the mass fraction at the same evolutionary stages considered in Figs. 8 and 9.

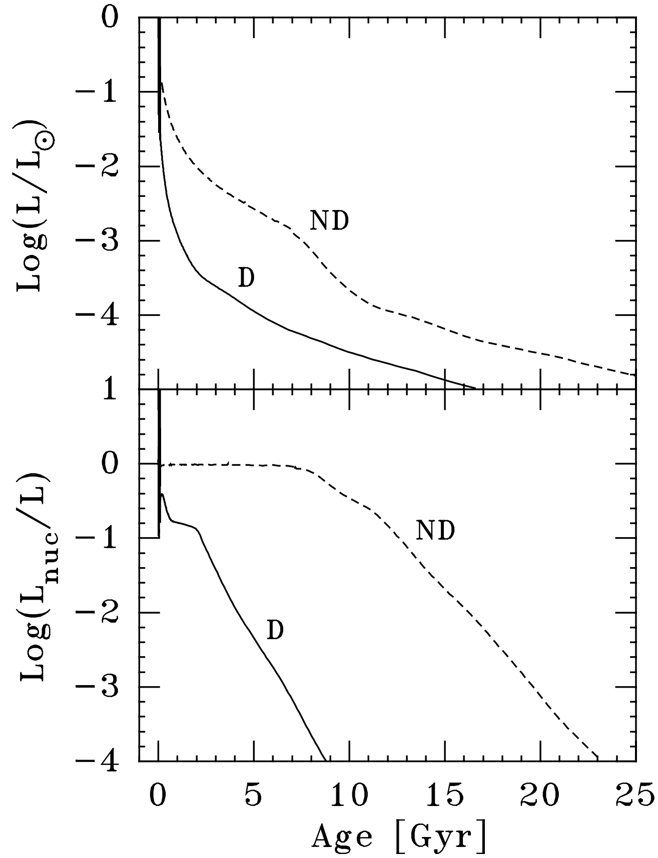

FIG. 11.—Surface and nuclear luminosity vs. age relationship for models with (*solid lines*) and without diffusion (*dashed lines*). Clearly, models with diffusion evolve much more rapidly, because the hydrogen envelope left is so thin that it is unable to support a appreciable amount of nuclear burning (*bottom panel*). This is in contrast with standard result without diffusion, for which hydrogen burning is the main energy source even at very low luminosities.

assertion on a more quantitative basis, we note that to reach log $L/L_\odot = -4$ models with diffusion require 5.3 Gyr, while models without diffusion need 13 Gyr (the latter is in agreement with the results of Driebe et al. 1998). It is important to note that the present results strongly alleviate the controversy raised by van Kerkwijk et al. (2000) about the large discrepancy between the ages of the pulsar PSR B1855+09 and its WD companion.

Now we are in a position to compare our present detailed results with those we previously presented (Benvenuto & Althaus 1998) based on the assumption of complete separation of hydrogen from helium. In that work we constructed several cooling models with different total amounts of hydrogen, handling that quantity as a free parameter. The corresponding cooling curves and fractions of nuclear luminosities are shown in Figure 12, where we include models with $M_H/M_* = 0$, 10^{-4}, 2×10^{-4}, 10^{-3}, and 2×10^{-3}. Because we started with an artificial model, the zero age is arbitrary, and in view of the numerical technique employed for this purpose it is impossible to make a direct comparison between the ages based on the old models and those presented in this paper. Thus, we are free to look for a fit between two curves by performing a horizontal displacement. It is immediately apparent that the curves corresponding to $M_H/M_* = 2 \times 10^{-3}$ show an excellent agreement with our new results. Notably, this value for

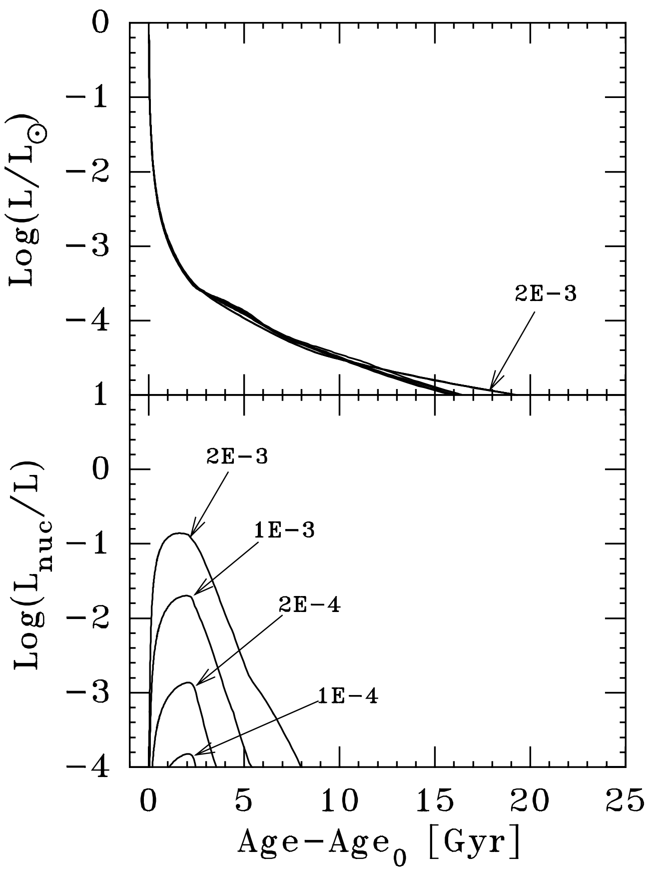

FIG. 12.—Same as Fig. 11, but for models taken from Benvenuto & Althaus (1998) for different values of the total hydrogen mass fraction. Because these sequences have been computed starting from artificial models, the time origin is arbitrary and is denoted as Age_0. Note that cooling curves (*top panel*) are very similar to those predicted by the detailed treatment of the problem in this paper.

M_H/M_* is very similar to the one found with our detailed diffusion calculation: $M_H/M_* = 1.7 \times 10^{-3}$. Thus, detailed calculations that include diffusion and start from a physically plausible model strongly support the results presented in Benvenuto & Althaus (1998).

4. DISCUSSION AND CONCLUSIONS

In this paper we have presented calculations of the evolution of a 0.242 $M_\odot$ object that finally becomes a helium white dwarf (WD). To this end, we have employed our stellar code, to which we have added a proper treatment for element diffusion. In particular, we have considered gravitational settling and chemical and thermal diffusion. We have constructed our initial models by abstracting mass from a 1 $M_\odot$ red giant branch star up to the moment at which it begins to evolve to the blue part of the H-R diagram. Each sequence, with and without diffusion, comprises about 30,000 models, each divided into about 2000 mesh points. Our results without diffusion are very similar to those presented in Driebe et al. (1998).

From the results presented in the previous section, it is clear that element diffusion introduces important changes in the internal structure of the star. In particular, for the present stellar mass value, models with diffusion undergo three thermonuclear flashes, whereas models without diffusion undergo only one. This fact has a large effect on the fraction of total hydrogen mass M_H/M_* left in the star (about 3 times as much hydrogen remains in models without diffusion) at the start of the final cooling track. Thus, while standard models have a very active hydrogen-burning zone down to very low luminosities (in fact, nuclear luminosity is very similar to the total photon luminosity of the star), models with diffusion have a much lower nuclear energy release. In the case of diffusion models, the star has to take energy from its relic thermal content, which makes the object cool significantly faster compared with the standard treatment.

The structure of the models belonging to the final cooling branch for the 0.242 $M_\odot$ helium WD strongly resembles the one we have assumed in our previous work on helium WDs with hydrogen envelopes (Benvenuto & Althaus 1998). In fact, if we assume $M_H/M_* = 2 \times 10^{-3}$ the agreement with the present results is excellent, not only regarding the cooling evolution but also with respect to the nuclear activity. In our opinion, the present results largely justify the assumptions we have made in our previous paper.

Our present detailed results indicate that, contrary to the assertions of Driebe et al. (1998) and Schönberner, Driebe, & Blöcker (2000), nuclear reactions are not an important ingredient in determining the age of the star during the final evolution. This conclusion is valid at least for the stellar mass value analyzed in this paper. Although the hydrogen mass fraction at the end of the computed stages (log $L/L_\odot = -5$) is very similar in sequences both with and without diffusion, there is a large difference in the total time spent in reaching such a low luminosity. While standard models without diffusion spend about 30 Gyr in reaching log $L/L_\odot = -5$, models with diffusion achieve this luminosity in about 15 Gyr.

As stated in § 1, previous models available in the literature (those in which hydrogen burning is dominant during most of the cooling stages) lead to a strong discrepancy between the ages of PSR B1855+09 and its helium WD companion. These ages should be, on very general grounds, the same. The results presented in this paper strongly indicate that such a discrepancy is a direct consequence of ignoring element diffusion in stellar model calculations. If we include it properly, ages naturally come into a nice agreement. Consequently, we do not have to invoke any ad hoc mass loss, as proposed by Schönberner et al. (2000), or exotic mechanisms to account for the observational situation of the binary system containing PSR B1855+09.

We want to emphasize that our results strongly resemble those presented some time ago by Iben & Tutukov (1986, hereafter IT86) for the case of a 0.296 $M_\odot$ model (i.e., slightly more massive than our model). They found two flashes and suspected that the second one was due to diffusion, but because they did not compute models without diffusion, they were not conclusive about this point. Here, by contrast, we confirm that diffusion is the reason for the occurrence of additional flashes. It is remarkable that IT86 also found that the final amount of hydrogen left in the WD is small enough to ultimately enforce a fast cooling. Although their model is more massive than ours, which prevents a quantitative comparison between the two sets of calculations, the role of hydrogen burning at the final cooling stages is the same as we have found. There is nevertheless an important difference between the work of IT86 and our study. IT86 considered that shortly after the second flash, when the model reaches its maximum radius, it again overfills its

Roche lobe and loses a tiny part of its stellar mass. Obviously, this mass belongs to the hydrogen envelope and has a large effect on the final value of M_H/M_* in the WD. This is ultimately responsible for the fast cooling occurring at late stages of evolution. The point to emphasize here is that we did not consider such a mass transfer during our calculations, and some words about this point are in order. If such a transfer actually occurs, M_H/M_* should drop below the values we presented above (eventually to zero). This would have an effect on the global structure of the WD. Such a situation may be considered in the frame of the models presented in Benvenuto & Althaus (1998). Apart from the differences in surface gravity and convection zones, for example, we do not expect an important change in the timescale of cooling. Note that in Figure 12 we have included cooling models with M_H/M_* from 1×10^{-3} to zero. All such models reach very low luminosities at essentially the same age. Thus, the occurrence of such a late mass transfer should not modify the conclusions about the age of the final WD model and its better agreement with the observations related to the ages of the PSR B1855+09 system.

In closing, let us discuss the effects of some of the approximations we have made in constructing the evolutionary sequences presented here. As stressed above, when the star ignites hydrogen in a flash fashion, the whole star suddenly changes its structure. For this reason, the standard assumption of hydrostatic equilibrium should be taken with caution during the postflash expansion. In fact, the inertial term in the equation of motion may not be entirely negligible. For instance, when the star returns to the red giant branch as a result of postflash expansion, the dynamical timescale becomes ≈ 0.03 yr and the evolutionary one of the order of 0.5 yr. Needless to say, the effects induced by the neglect of the inertial term should be confined to modifying the fastest part of the evolutionary tracks (loops) but not the global properties of the star. More dramatic seems to be the hypothesis of instantaneous mixing in convective zones. It is clear that a more physically plausible treatment of the mixing (like that presented by Ventura et al. 1998) should bring less hydrogen from outer layers to the hot layers of the star, diminishing the effectiveness of flashes in burning hydrogen. Relaxation of the instantaneous mixing hypothesis may eventually lead to evolutionary models with a larger number of thermonuclear flashes. With respect to our employment of gray atmospheres, this is expected to affect only very late times of evolution, as discussed by Hansen (1999) and Salaris et al. (2000). We shall (hope to be able to) relax these simplifying assumptions in the near future.

In a forthcoming paper we shall explore the consequences of element diffusion in the whole range of masses expected for helium WDs.

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