Mon. Not. R. Astron. Soc. **332,** 399–408 (2002)

Time-dependent diffusion in pulsating white dwarf stars: asteroseismology of G117-B15A

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Accepted 2001 December 21. Received 2001 October 19; in original form 2001 July 30

ABSTRACT

We study the structural characteristic of the variable DA white dwarf G117-B15A by applying the methods of asteroseismology. For such a purpose, we construct white dwarf evolutionary models considering a detailed and up-to-date physical description as well as several processes responsible for the occurrence of element diffusion. We have considered several thicknesses for the outermost hydrogen layer, whereas for the inner helium-, carbon- and oxygen-rich layers we considered realistic profiles predicted by calculations of the white dwarf progenitor evolution. The stellar masses we have analysed cover the mass range of $0.50 \leq M_*/\mathrm{M}_\odot \leq 0.60$.

The evolution of each of the considered model sequences was followed down to very low effective temperatures; in particular, from 12 500 K on we computed the dipolar, linear, adiabatic oscillations with radial order $k = 1, \ldots, 4$. We find that asteroseismological results are not univocal regarding mode identification for the case of G117-B15A. However, our asteroseismological results are compatible with spectroscopic data only if the observed periods of 215.2, 271.0 and 304.4 s are due to dipolar modes with $k = 2, 3, 4$, respectively.

Our calculations indicate that the best fit to the observed period pattern of G117-B15A corresponds to a DA white dwarf structure with a stellar mass of $0.525\,\mathrm{M}_\odot$, with a hydrogen mass fraction $\log(M_\mathrm{H}/M_*) \gtrsim -3.83$ at an effective temperature $T_\mathrm{eff} \approx 11\,800$ K. The value of the stellar mass is consistent with that obtained spectroscopically by Koester & Allard.

Key words: stars: evolution – stars: individual: G117-B15A – stars: interiors – stars: oscillations – white dwarfs.

1 INTRODUCTION

Asteroseismology has become a powerful method to disentangle the internal structure and evolution of stars by means of the study of their oscillatory pattern. This technique, very sophisticated in the case of the Sun, has also undergone a strong development in other stars, in particular in variable white dwarf (WD) stars (see e.g. Brown & Gilliland 1994; Gautschy & Saio 1995, 1996).

Pulsating WDs show multiperiodic luminosity variations in three ranges of effective temperatures (T_eff) corresponding to the currently called DOV, DBV and DAV (see e.g. the review by Winget 1988). Of interest in this work are the DAVs (hydrogen-dominated atmospheres), or ZZ Ceti, which pulsate in the instability strip corresponding to $11\,000 \leq T_\mathrm{eff} \leq 13\,000$ K. The periodicities in the light curves of pulsating WDs are naturally explained in terms of non-radial g-modes of low harmonic degree ($\ell \leq 2$), driven by the 'κ mechanism' working in a partial ionization region near the stellar surface (Dolez & Vauclair 1981; Winget et al. 1982). The periods (P) are found within a range of $100 \leq P \leq 1200$ s and photometric amplitudes reach up to 0.30 mag.

Most asteroseismological studies performed on WDs to date rely on stellar models constructed under some simplifying hypotheses. One of the most relevant hypotheses is related to the profile of the internal chemical composition in the interface zones. In such regions, the equilibrium diffusion in the trace element approximation has been widely employed to infer the profile of the chemical distribution (see Tassoul, Fontaine & Winget 1990). The main motivation for considering this approximation is to avoid

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the solution of time-dependent element diffusion as the WD evolves. In the frame of such an approximation, the profile of the interface region is very simple: its functional form is a power law. The transition zone is separated into two parts: an upper one in which one element is dominant and the other is considered as a trace, and a lower region in which the role of the respective elements is reversed. Because these two power-law solutions are matched for fulfilling the condition of conservation of mass of each element, a discontinuity in the derivative occurs just at the matching point. The exponent of the power-law solution is directly related to the state of ionization of the stellar plasma. Thus, the structure of the whole interface zone, in the frame of this standard treatment, can be modified only if the plasma suffers from a modification in the state of ionization as a result of stellar evolution. Calculations of asteroseismology of WDs in the frame of such a standard treatment for the chemical interfaces are those of e.g. Bradley (1996, 1998, 2001), Bradley & Winget (1994), Brassard et al. (1991, 1992a,b), Fontaine et al. (1992), Montgomery & Winget (1999), Metcalfe, Nather & Winget (2000), Metcalfe, Winget & Charbonneau (2001) and Montgomery, Metcalfe & Winget (2001).

From an evolutionary point of view, the shape of the chemical interfaces may not be critical,[1] but in pulsational studies they provide a non-negligible contribution to the shape of the Ledoux term of the Brunt–Väisälä frequency (Brassard et al. 1991). Thus, we can, in principle, expect differences between studies with equilibrium diffusion and those in which other more physically sound treatment is performed. This is a very important point to be made in connection with the aim of the present work.

Since the pioneering work of Iben & MacDonald (1985), we know that element diffusion modifies the chemical abundance distribution within a WD star even during evolutionary stages corresponding to the ZZ Ceti domain (see Iben & MacDonald 1985, particularly their fig. 4). Few calculations exist in the literature in which the evolution of WDs is addressed in a self-consistent way with time-dependent element diffusion. Amongst them, we mention the study of MacDonald, Hernanz & José (1998) aimed at studying the carbon pollution in cool WDs. Also, Dehner & Kawaler (1995) used non-equilibrium diffusion together with evolutionary calculations to study WDs with helium envelopes. Finally, Althaus, Serenelli & Benvenuto (2001) have recently shown that diffusion induces the occurrence of thermonuclear flashes in helium-core WDs, causing the evolution of such WDs to occur on time-scales significatively shorter than predicted by models without diffusion. This has been particularly important in solving the discrepancy (Van Kerkwijk et al. 2000) about the age of binary systems containing a millisecond pulsar and a helium WD.

As far as we are aware, the only work aimed at exploring the role played by diffusion in the period and its rate of change of g-modes in pulsating DA WDs is that of Córsico et al. (2002). In particular, the authors found that the differences in the shape of the chemical profiles at the interface zones induce appreciable changes in the periods, as compared to the case of equilibrium diffusion in the trace element approximation. Also, there are noticeable changes in the period derivative, which are due in part to the evolution of the chemical profile during the cooling of the WD across the ZZ Ceti instability strip.

Since some time ago there have been several works available in the literature in which the observed period structure of a particular object is fitted to theoretical predictions. Such works show that, in principle, information about the stellar mass and the stratified outer layer structure can be inferred (see e.g. Winget et al. 1991; Fontaine et al. 1992; Pfeiffer et al. 1996; Bradley & Winget 1994; Bradley & Kleinman 1997; Bradley 1998, 2001). As these studies are based on a simplified treatment of diffusion, we believe that it is worth revisiting this problem on the basis of more detailed models of WD structure. In particular, in view of the results found in Córsico et al. (2002), we expect to find differences in the period fitting to a particular object, as compared with the situation in which the standard treatment is used. The only way to find how important such differences can be is by performing a detailed asteroseismological study choosing a well-studied object.

We consider G117-B15A as an optimal target for our study. G117-B15A is an otherwise typical DA WD, the variability of which was discovered by McGraw & Robinson (1976) and, since then, it has been monitored continuously. The mass and in particular the effective temperature of this star have been the subject of numerous spectroscopic re-determinations (see Gautschy, Ludwig & Freytag 1996 for a summary). In particular, values of $0.59\,M_{\odot}$ and 11 620 K, respectively, have been derived by Bergeron et al. (1995b). More recently, Koester & Allard (2000; hereafter KA) have suggested a somewhat lower value for the mass of $0.53\,M_{\odot}$ and a higher effective temperature of $T_{\rm eff} = 11\,900 \pm 140$ K.

G117-B15A has periods of oscillation of 215.2, 271 and 304.4 s (Kepler et al. 1982). Notably, for the 215.2-s mode it has been possible to find a value of its temporal derivative (Kepler et al. 2000) to be $\dot{P} = (2.3 \pm 1.4) \times 10^{-15}\,{\rm s\,s^{-1}}$. Interestingly, the 215.2-s mode present in G117-B15A is the most stable optically registered oscillation with a stability comparable to that of the most stable millisecond pulsars. As G117-B15A is a well know oscillator, it has motivated the interest of several researchers in its internal structure. For example, Bradley (1998) has found the best fit to the period pattern with a model of $\approx 0.6\,M_{\odot}$, and depending on the identification of the modes (see below for an explanation of its meaning), the favoured value for the hydrogen envelope mass fraction $M_{\rm H}/M_{*}$ is $-7 \lesssim \log(M_{\rm H}/M_{*}) \lesssim -6$ or $-5 \lesssim \log(M_{\rm H}/M_{*}) \lesssim -4$, and $\log(M_{\rm He}/M_{*}) = -2$. More recently, Córsico et al. (2001) have found, on the basis of WD models in which diffusion is neglected, the best fit to the period pattern with a $0.55\,M_{\odot}$ WD model with a carbon–oxygen interior, $\log(M_{\rm He}/M_{*}) = -2$ and $\log(M_{\rm H}/M_{*}) = -4$ as predicted by stellar evolution. Notably, such a fitting is in nice agreement with one of those proposed by Bradley (1998).

It is the aim of this work to perform a fitting to the period structure present in G117-B15A by computing the non-radial eigenmodes in the frame of linear and adiabatic approximation, and evolutionary WD models in which time-dependent element diffusion is properly accounted for. About the way we shall perform such a fitting, some words are in order. In handling models like those we shall employ it is quite obvious that we are not in a position to employ techniques like those used in the so-called genetic algorithm (see its application for the case of the DBV GD358 in Metcalfe et al. 2000). In the context of ZZ Ceti stars, a similar approach, though to a lesser extent, has been repeatedly employed by numerous investigations (e.g. the complete and detailed study performed by Bradley 1998, 2001). Up to now, the shape and thickness of the relevant chemical interfaces are usually treated as free parameters. However, a more physically sound treatment can be performed when account is made of time-dependent element diffusion in evolutionary models of WDs. This

[1] An obvious and important exception is when the tail of chemical profiles are subject to nuclear burning.

aspect of WD evolution is one of the most important when an attempt is made to compare observations with theoretical expectations from pulsating WDs. Unfortunately, even if a pre-WD model is fixed, there are some uncertainties that prevent us from predicting a whole definite internal structure (e.g. the treatment of convection, the rate of the critical nuclear reaction $^{12}C(\alpha,\gamma)^{16}O$, wind mass loss episodes, etc.). This is especially true for M_H/M_*. Thus, in view of these facts, we shall consider the value of M_H/M_* as a free parameter. We assumed $\log(M_{He}/M_*) = -2$ throughout this paper.

The remainder of the paper is organized as follows. In Section 2 we discuss the physical ingredients we have employed together with the computational strategy we have employed. Section 3 is devoted to presenting the details of the evolutionary models that we have constructed down to the conditions relevant for the WD we are investigating. In Section 4 we present the asteroseismological analysis we performed for the particular case of G117-B15A. Section 5 is devoted to the discussion of the results we have found, and finally in Section 6 we give some concluding remarks and insights for future investigations.

2 DETAILS OF THE COMPUTATIONS

For this work we have employed the same code and physical ingredients as in Córsico & Benvenuto (2002) and Córsico et al. (2001, 2002). In particular, for the diffusion processes we considered gravitational settling, chemical and thermal diffusion as in Althaus & Benvenuto (2000).

In our code, evolutionary and pulsational calculations are performed in a fairly automatic way. After selecting a starting stellar model we choose an interval in P and $T_{\rm eff}$. The evolutionary code computes the model cooling until the hot edge of the $T_{\rm eff}$-interval is reached. Then, the program calls the set of pulsation routines beginning the scan for modes. When a mode is found, the code generates an approximate solution which is iteratively improved to convergence (of the eigenvalue and the eigenfunctions simultaneously) and stored. This procedure is repeated until the period interval is covered. Then, the evolutionary code generates the next stellar model and calls pulsation routines again. Now, the previously stored modes are taken as initial approximation to the modes of the present model and iterated to convergence. Such a procedure is automatically repeated for all evolutionary models inside the chosen $T_{\rm eff}$-interval. The computational strategy described above has been successfully applied in fitting the observed periods of G117-B15A to impose constraints on the mass of axions (Córsico et al. 2001) in the frame of WD models less detailed than those we shall employ in the present work. We refer the reader to Córsico & Benvenuto (2002) and Córsico et al. (2001, 2002) and references therein for further details. Let us quote that most of our evolutionary models have been divided in more than 2000 mesh points, whereas for mode calculations we employed 5000 mesh points.

In order to start our evolutionary calculation we have employed an artificial heating technique detailed in Althaus & Benvenuto (2000). Let us quote that in this technique, the stellar model is forced to undergo some unphysical evolution by introducing an artificial energy release that, when the star gets bright enough, is switched off smoothly. Then, the star relaxes to the physical cooling branch after a few tens of models.

In employing this artificial technique, we must be very careful to avoid generating an artificial chemical profile. Thus, in order to verify that the star not only relaxes to the physical cooling branch but *simultaneously* relaxes to a correct internal chemical profile, we have employed, for a fixed $\log(M_H/M_*)$ value, different shapes of the initial hydrogen and helium profiles. We found that, if the initial model is far enough away from the evolutionary stage we are interested in, the fine details of the initial profile are irrelevant because diffusion largely evolves it to a well defined one. As a matter of fact, the structure and the chemical profiles are physically plausible far before the star attains the effective temperature range relevant for G117-B15A.

Table 1. Values of $\log(M_H/M_*)$ for our computed models at $T_{\rm eff} = 12\,500$ K.

0.50 $M_\odot$	0.55 $M_\odot$	0.60 $M_\odot$	0.525 $M_\odot$	0.5375 $M_\odot$
−3.815	−3.862	−3.941	−3.831	−3.843
−4.193	−4.224	−4.249	−4.198	...
−4.679	−4.684	−4.692	...	...
−5.175	−5.177	−5.180	...	...
−5.671	−5.671	−5.672	...	...
−6.160	−6.158	−6.156	−6.159	...
−6.640	−6.633	−6.700	−6.634	...
−7.071	−7.047	−7.028	...	...
−7.423	−7.385	−7.349	...	...

A better approach for constructing pulsating WD models would obviously be to start computing evolutionary models from much earlier in the star history. This possibility, better from a physical point of view, again would require an enormous amount of computing time for a definite sequence. Thus trying to fit a period pattern in such a way is, in our opinion, far beyond what seems reasonable at the present state of the art. In any case, it would be interesting to explore the pulsational properties of models resulting from a full evolutionary calculation. We plan to present such a study in a future paper.

To be specific, we have started the calculations with models of $\log(L/L_\odot) \approx 2.05$, $\log T_{\rm eff} \approx 4.46$ and masses of 0.50, 0.55 and 0.60 $M_\odot$ (in performing the fitting of the models to observations we have also computed supplementary models of 0.525 and 0.5375 $M_\odot$). The $\log M_H/M_*$ values present in the star after evolving along the cooling branch down to $T_{\rm eff} = 12\,500$ K are those given in Table 1. At the starting conditions we have assumed the Salaris et al. (1997) core composition, whilst for the outer layers the CNO abundances were those corresponding to the pre-WD evolution. In order to get different values of M_H/M_*, we have started from the model with the largest M_H/M_* and simply replaced ^{1}H by ^{4}He at the base of the hydrogen envelope. Let us quote that the initial model has internal temperatures low enough to prevent the occurrence of nuclear reactions up to the moment at which the WD model has relaxed to the physical branch. We should mention that models become physically adequate (after the unphysical transitory) for describing a WD at $\log(L/L_\odot) \approx 1.4$, $\log T_{\rm eff} \approx 4.83$, far from the conditions relevant for DAV WDs. Also, the values of $\log(M_H/M_*)$ at the instability strip are somewhat lower than those included in Table 1, simply because nuclear reactions are still operating at such evolutionary stages. However, this effect is noticeable only for the case of the thickest hydrogen envelopes on the most massive objects considered here.

To our notice, this is the first opportunity in which carbon–oxygen, intermediate-mass WD models with a set of values in M_H/M_* are evolved in the frame of time-dependent element diffusion. Prior to this work, the kind of calculations carried out

here were performed only for the values of M_H/M_* predicted by stellar evolution theory (Iben & MacDonald 1985, 1986).

3 EVOLUTIONARY RESULTS

For each initial model, we have computed its evolution down to the conditions corresponding to the DAV instability strip. In doing so, we computed about 10 000 models for each sequence. Perhaps the most relevant characteristic of the models for the kind of study we are performing here is the internal chemical profile. Results representative for the case of the models corresponding to the DAV instability strip are presented in Fig. 1. In this figure we depict the internal chemical profiles for a 0.55 $M_\odot$ WD model at $\log T_{\rm eff} \approx 4.096$ for $\log(M_H/M_*) = -3.862$. The adopted effective temperature is slightly higher than the last estimation for G117-B15A ($\log T_{\rm eff} \approx 4.076$). Thus, these profiles should be representative of the conditions expected for the interior of G117-B15A in the frame of our set of models.

Let us discuss the main characteristics of these profiles. Before discussing them, we have to warn the reader that the way we have modified the value of M_H/M_* is ad hoc. In fact, it is not clear at present if there exists, for a fixed stellar mass value, an evolutionary mechanism capable to produce a distribution of M_H/M_* values differing from each other by orders of magnitude. In addition, the outward tail of the distribution of initial heavy element abundances may be different from the results we shall present below. However, we should remark that the profile of the light elements present in the outer parts of the models should be essentially correct.

With regard to the chemical profiles of our models, we want to mention that for $\log(1 - M_r/M_*) \gtrsim -1.6$ the carbon–oxygen distribution is that predicted by pre-WD evolutionary calculations and it is the same for all of our evolutionary sequences. However, this is *not* the case for the chemical structure of the outer layers, which are, in turn, crucial for the pulsational properties of the models. The main variations are related to the whole distribution of ^{1}H, ^{3}He and the outwards tail of ^{4}He. While a variation in the assumed value of M_H/M_* obviously implies a modification in the profiles, we find a very noticeable variation in the slope of the profiles for the isotopes present in the outer layers of our WD models. In the model with $\log(M_H/M_*) = -3.815$, the large slope in the tail of the hydrogen distribution is remarkable. In spite of this steep profile, degeneracy prevents hydrogen from diffusing further inwards. In models with a smaller hydrogen content, the tail of the hydrogen distribution is smoother. Note also that the shape of the tail of the hydrogen distribution at the evolutionary conditions relevant for DAV WDs is strongly dependent on the value of the assumed M_H/M_*.

Regarding the tail of the ^{4}He distribution, this is largely due to gravitational settling that forces ^{4}He to sink down. In particular our models also present a zone in which the chemical composition is largely dominated by helium. Also, it is of interest the distribution of ^{3}He. It is clearly noticeable that most of the ^{3}He is located in stellar layers at which ^{1}H and ^{4}He are present. This is a consequence of the interplay between nuclear reactions that form ^{3}He by means of the first two reactions of the proton–proton I chain (Clayton 1968),

$$^1\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^2\mathrm{H} + e^+ + \nu$$

$$^2\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^3\mathrm{He},$$

and gravitational settling. Notice that the outer tail of the ^{3}He distribution is similar to the corresponding ^{4}He which is, in part, due to the operation of the reaction

$$^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2\,{}^1\mathrm{H}.$$

As the chemical profile of the hydrogen in surface layers is one of

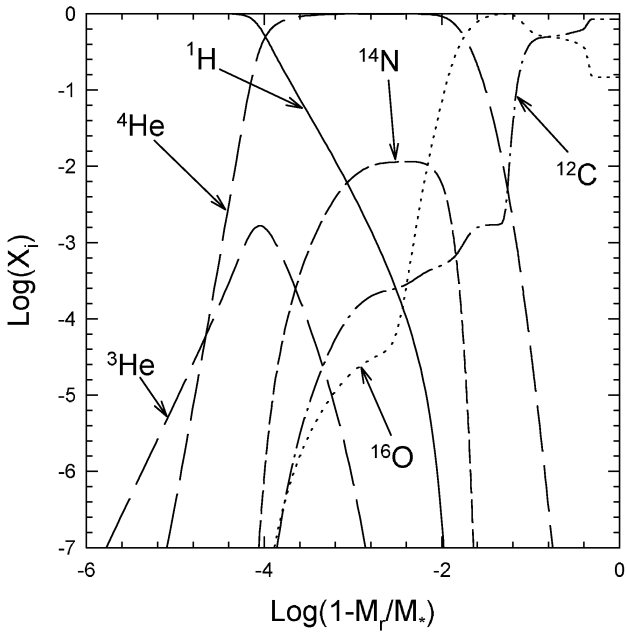

Figure 1. The internal chemical profile of a 0.550-$M_\odot$ WD model for ^{1}H, ^{3}He, ^{4}He, ^{12}C, ^{14}N and ^{16}O, for the case of $\log(M_H/M_*) = -3.862$ at $\log T_{\rm eff} = 4.096$. For more details, see text.

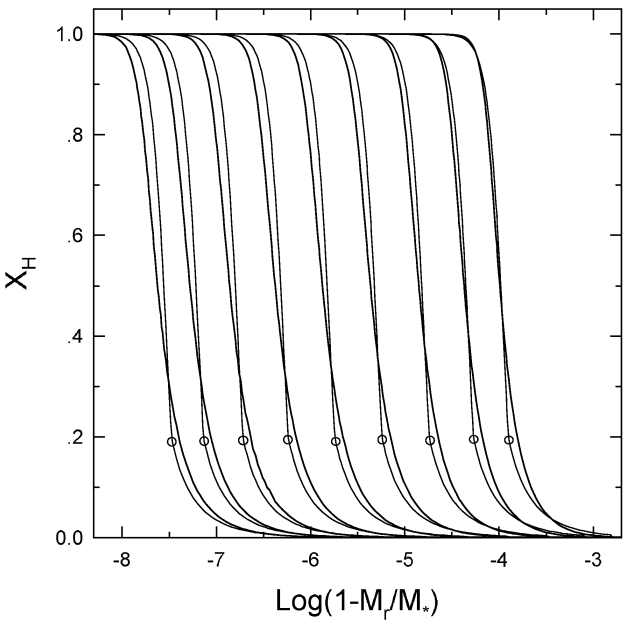

Figure 2. The profile of the bottom of the hydrogen envelope for our suite of 0.55-$M_\odot$ models at $T_{\rm eff} = 12500$ K, represented by solid lines. For the corresponding values of M_H/M_*, see Table 1. Thin lines represent the profiles corresponding to the standard treatment of equilibrium diffusion in the trace element approximation for the same M_H/M_* values. Notice that while for thick hydrogen envelopes the profiles are rather similar, there appear large differences in the case of thin envelopes. Circles denote the change of slope in the profiles calculated with equilibrium diffusion. For more details, see text.

the key ingredients in determining the g-modes of DAV WDs, we show in Fig. 2 the base of the hydrogen envelopes for our suite of $0.55\,M_\odot$ models at the ZZ Ceti domain. In addition, we show the profiles predicted by the standard treatment of equilibrium diffusion in the trace element approximation for the same stellar mass and M_H/M_* values. It is clear that time-dependent element diffusion predicts hydrogen profiles that are somewhat different from those given by the standard treatment. In the standard treatment, the slope of the profile is determined solely by the state of ionization of the plasma (see equations 26–29 of Tassoul et al. 1990). Thus, if the state of ionization is the same, the slope should remain essentially unaltered with further evolution. We have verified that, even in the case of the thinnest hydrogen envelopes we have considered, the base of the hydrogen envelope is located at layers with temperatures above 10^6 K, which ensures complete ionization for hydrogen and helium and this is so even at the effective temperatures corresponding to the DAV instability strip. Also, notice that the sudden change of the slope at the matching point of the two solutions of the standard treatment (denoted by circles along the curves) has a direct consequence on the enhancement of the trapping of some modes in the hydrogen envelope (see e.g. Brassard et al. 1992a; Bradley 1996). Thus, our results show the necessity of allowing for a more detailed treatment of the chemical evolution at performing asteroseismology of WDs.

Now, let us discuss the impact of our chemical profiles on the Brunt–Väisälä frequency, a key quantity entering as a coefficient in the equations of non-radial pulsations. This is defined as (Unno et al. 1989)

$$N^2 = g\left(\frac{1}{\Gamma_1}\frac{\mathrm{d}\ln P}{\mathrm{d}r} - \frac{\mathrm{d}\ln\rho}{\mathrm{d}r}\right) \tag{1}$$

where all the symbols have their usual meaning[2] and Γ_1 is the first adiabatic exponent. However, in order to avoid numerical noise, we use the expression given by Brassard et al. (1991):

$$N^2 = \frac{g^2\rho}{P}\frac{\chi_T}{\chi_\rho}[\nabla_{ad} - \nabla + B]. \tag{2}$$

The Ledoux term B, for the case of a M-component plasma, is given by

$$B = -\frac{1}{\chi_T}\sum_{i=1}^{M-1}\chi_{X_i}\frac{\mathrm{d}\ln X_i}{\mathrm{d}\ln P}. \tag{3}$$

where

$$\chi_\rho = \left(\frac{\partial\ln P}{\partial\ln\rho}\right)_{T,\{X_i\}}$$

$$\chi_T = \left(\frac{\partial\ln P}{\partial\ln T}\right)_{\rho,\{X_i\}}$$

$$\chi_{X_i} = \left(\frac{\partial\ln P}{\partial\ln X_i}\right)_{\rho,T,\{X_{j\neq i}\}}.$$

In Fig. 3 we show the squared Brunt–Väisälä frequency and the Ledoux term for models of $0.55\,M_\odot$ with $\log T_{\rm eff} \approx 4.096$ and all the considered values of M_H/M_*. We also show in an inset the Brunt–Väisälä frequency at the hydrogen–helium interface for the case of the thickest hydrogen envelope according to our computations and to the prediction of equilibrium diffusion (thick and thin lines, respectively). In order to show the correspondence of the structure of these functions with the internal chemical profiles of the star, we have also included the curves corresponding to the abundances of ^{4}He and ^{12}C.

The Ledoux term B has a structure that remains largely invariant for the stellar core under changes in M_H/M_*. From the centre outwards, the three peaks are due to steep slopes in the carbon profile. The first two are due to the structure of the carbon–oxygen interface, whereas the other is due to the helium–carbon interface. In contrast to the internal behaviour of the Ledoux term, the other peak, due to the hydrogen–helium transition, is largely modified, not only in position but also in its height. Notice that the lower the value of M_H/M_*, the thinner the peak (see Brassard et al. 1991 and Tassoul et al. 1990 for comparison of the shape of the B term and Bradley 1996 for the profile of the Brunt–Väisälä frequency).

It is worthwhile to comment on the fact that because of our realistic chemical profiles, we would expect to find differences in

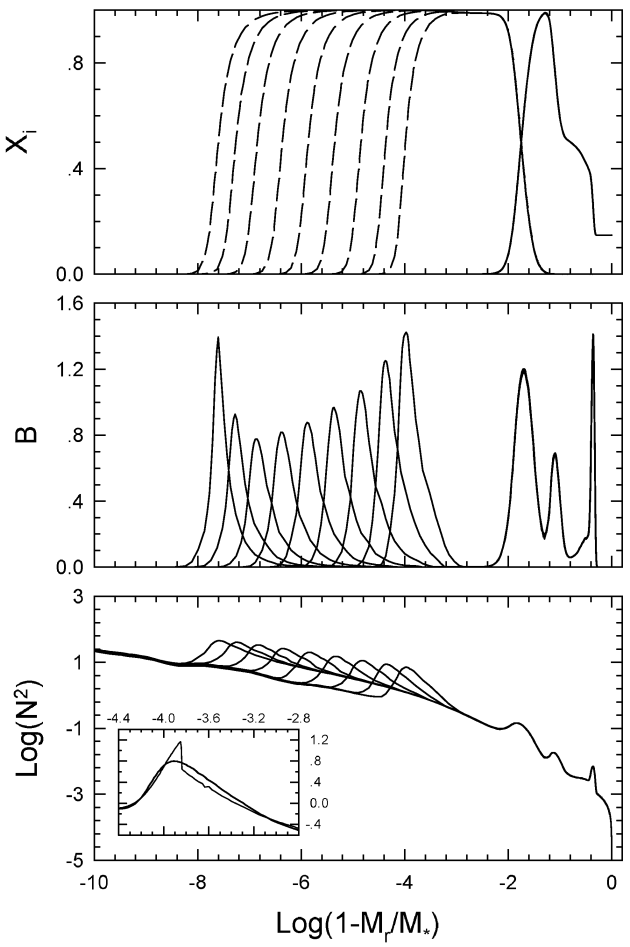

Figure 3. Some of the main characteristics of our models for the case of a mass of $0.55\,M_\odot$. Upper panel: carbon (solid lines) and helium (long-dashed lines) profiles. Middle panel: the Ledoux term of the Brunt–Väisälä frequency. Lower panel: the logarithm of the squared Brunt–Väisälä frequency. From the centre outwards, the three peaks of the Ledoux term B are due to steep slopes in the carbon profile. The first two are due to the structure of the carbon–oxygen interface whereas the other is due to the helium–carbon interface. The inset in lower panel shows the behaviour of the Brunt–Väisälä frequency in the case of time-dependent diffusion and according to the predictions of equilibrium diffusion for our sequence with the thickest hydrogen envelope at the hydrogen–helium interface (thick and thin lines, respectively). For more details, see text.

[2] In this section P stands for the pressure, but in the rest of the paper it stands for the period of oscillation.

the mode trapping properties as compared with previous studies. This important issue surely deserves a careful exploration, which will be reported in a subsequent communication.

4 ASTEROSEISMOLOGICAL RESULTS

An important aspect for the present work is the so-called mode identification. That is, the identification of the ℓ and k values corresponding to each observed period. In the case of G117-B15A, the results of Robinson et al. (1995) indicate that the 215.2-s period corresponds to a dipolar mode ($\ell = 1$). Following the work by Bradley (1998), we shall assume that the other two modes cited above are also dipolar (Brassard et al. 1993; Fontaine & Brassard 1994), and also that the other periodicities present in the light curve of the star are not associated with actual eigenmodes but are actually due to non-linear effects in the envelope (Brassard et al. 1993). With regard to the radial order of the modes, there exist two possible identifications. Following Clemens (1994) the observed periods are dipolar modes with $k = 2$ (215.2 s), $k = 3$ (271 s) and $k = 4$ (304.4 s). On the other hand, Fontaine, Brassard & Wesemael (1994) identified these modes as dipolar with $k = 1$ (215.2 s), $k = 2$ (271 s) and $k = 3$ (304.4 s). According to these facts, in computing the g-modes of our WD models, we have considered (as previously mentioned) only $\ell = 1$ and radial order $k = 1, \ldots, 4$. Higher radial orders correspond, for our stellar mass values, to periods long in excess compared to those observed and thus are not considered here.

In order to look for a fit to the observations, we shall consider a function ψ defined as

$$\psi = \log \sum_{i=1}^{3} \left[1 - \frac{P_i^{\rm C}}{P_i^{\rm O}} \right]^2 , \qquad (4)$$

where $P_i^{\rm O}$ and $P_i^{\rm C}$ are the ith observed and calculated periods, respectively. For the case of G117-B15A, $P_i^{\rm O} = 215.2$, 271 and 304.4 s. As a result of the pulsational calculations we have the values of the periods for $k = 1, \ldots, 4$. Then, assuming the two reasonable possible identifications of the observed modes (i.e. $P_i^{\rm O}$ corresponding to $k = 1, 2, 3$, hereafter case A, and $P_i^{\rm O}$ corresponding to $k = 2, 3, 4$, hereafter case B) we can compute the function ψ in terms of the effective temperature of the star for each of the considered sequences. Obviously in the case that the fitting were exact, $\psi \rightarrow -\infty$. Thus the criterion we shall assume in selecting the best model for representing G117-B15A is simply to look for the minima of the ψ function considering the whole set of models. In such a way we shall be able to determine not only the internal structure and mass of G117-B15A, but also its *asteroseismological* effective temperature.

In Fig. 4 we show ψ as a function of the effective temperature for the case of the set of models with 0.50 $M_\odot$. In this figure the left-hand panel labelled as 'A' corresponds to the identification of the observed modes with $k = 1, 2, 3$ whereas the right-hand panel 'B' corresponds to the other identification of modes we shall consider: $k = 2, 3, 4$ modes. As it has been found by Bradley (1998), for cases A and B the best fits to observations are found for models with very different values of $M_{\rm H}/M_*$. While assuming case A, we find the best fit with rather thin hydrogen envelopes ($\log(M_{\rm H}/M_*) = -6.16$) and in the case B the best fit is found for the case of the thickest considered hydrogen envelope: $\log(M_{\rm H}/M_*) = -3.81$. It is remarkable that the minimum found for case A is a very deep one, corresponding to a mass and effective temperature ($T_{\rm eff} \approx 11\,400$ K) rather lower than those corresponding to the last determination of these parameters for G117-B15A.

If we increase the stellar mass to 0.55 $M_\odot$, the behaviour of the function ψ is largely changed (see Fig. 5). Despite that in case A the best fit is for $\log(M_{\rm H}/M_*) = -6.63$, it corresponds however to an effective temperatures far lower than that suggested by KA (and outside the DAV instability strip). By contrast, in case B the best fit is again provided by the model with the thickest hydrogen envelope, but again it occurs at effective temperatures somewhat too low.

Finally, in Fig. 6 we show the behaviour of function ψ for the case of the models constructed assuming a stellar mass value of 0.60 $M_\odot$. It is clear that in both cases of mode identifications the

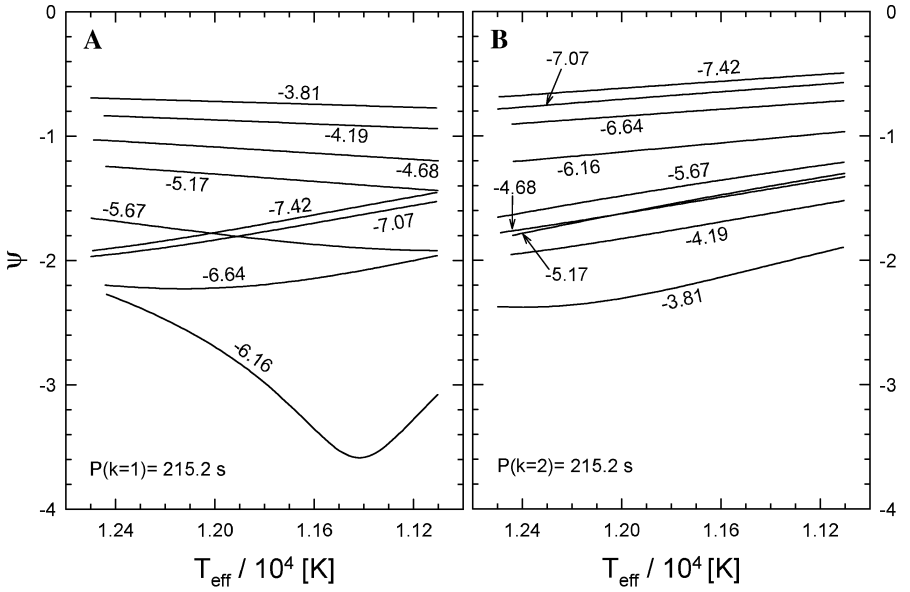

Figure 4. The value of ψ as a function of the effective temperature of the star for the case of 0.50-$M_\odot$ WD models. Curves are labelled with the respective value of $\log(M_{\rm H}/M_*)$. A and B stand for the identification of the observed modes with $k = 1, 2, 3$ and $k = 2, 3, 4$, respectively. Notice the very deep minimum of ψ for case A at $T_{\rm eff} = 11\,400$ K.

fits are far poorer than those for the other, lower mass values. Thus, the results presented up to this point strongly suggest that the asteroseismological mass of G117-B15A should be in between 0.50 and 0.55 $M_\odot$. Moreover, the only viable mass for the case A seems to be 0.50 $M_\odot$, because larger mass values give rise to poorer fits.

Let us analyse the results corresponding to case B more carefully, centering our attention in models with the thickest hydrogen envelopes. From Fig. 4(b), it can be noticed that in the case of the 0.50-$M_\odot$ model the fit is better for increasing effective temperatures. However, notice (Fig. 5b) that for the 0.55-$M_\odot$ model the situation is just the opposite: the fit improves for lower effective temperatures. Guided by this fact, a minimum in ψ can be expected for some intermediate value of the stellar mass. In order to explore this possibility we computed models with 0.525 $M_\odot$, restricting ourselves to values of M_H/M_* similar to those that provided good fits (see Table 1). For this mass value, the identification of modes in case A shows a good fit for $\log(M_H/M_*) = -6.63$ at $T_{\rm eff} \approx 11400$ while case B presents an interesting minimum at stellar mass and effective temperature values very similar to those determined by KA for this object by spectroscopic analysis. Finally, in order to perform a finer analysis, we compute a sequence of models with 0.5375 $M_\odot$ and a thick hydrogen envelope (see Table 1). The results for the whole set of masses and thickest hydrogen envelopes for case B is shown in Fig. 7. The labels of each curve correspond to the values of $\log(M_H/M_*)$ presented in Table 1.

The results presented in this section indicate that, if we expect

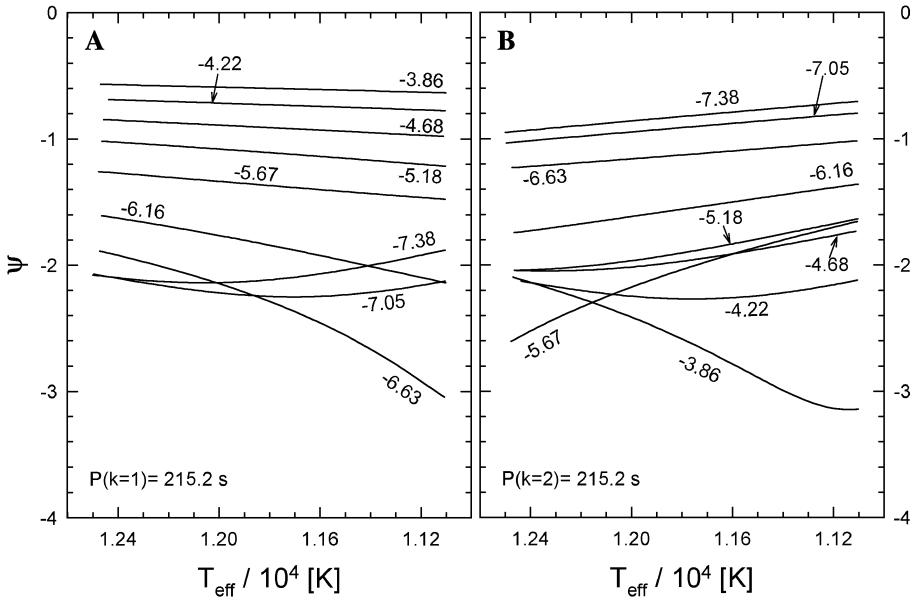

Figure 5. Same as Fig. 4 but for the case of 0.55-$M_\odot$ WD models. For this value of the stellar mass, minima corresponding to the two mode identifications have moved to the edges of the considered interval in effective temperatures.

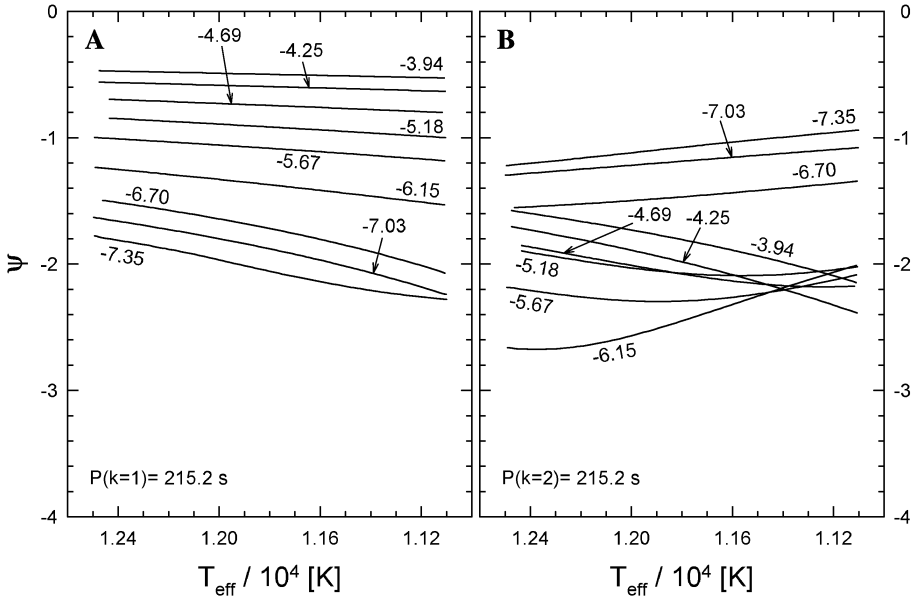

Figure 6. Same as Fig. 4 but for the case of 0.60-$M_\odot$ WD models. For this value of the stellar mass the values of ψ are larger than those found for the previously considered masses. Thus, the mass of G117-B15A should be lower than 0.60 $M_\odot$.

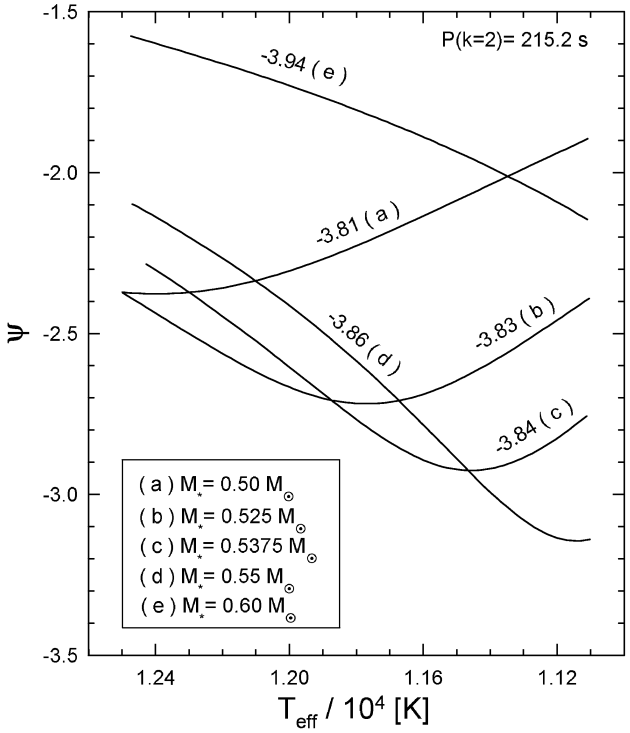

Figure 7. The value of ψ as a function of the effective temperature of the star assuming case B for the largest values of $\log(M_{\rm H}/M_*)$. Notice that if we assume the effective temperature to be in the range of the determination of Koester & Allard (2000), then, we naturally find that the mass of the object should be $0.525\,{\rm M}_\odot$ and its hydrogen mass fraction should be $\log(M_{\rm H}/M_*) = -3.83$.

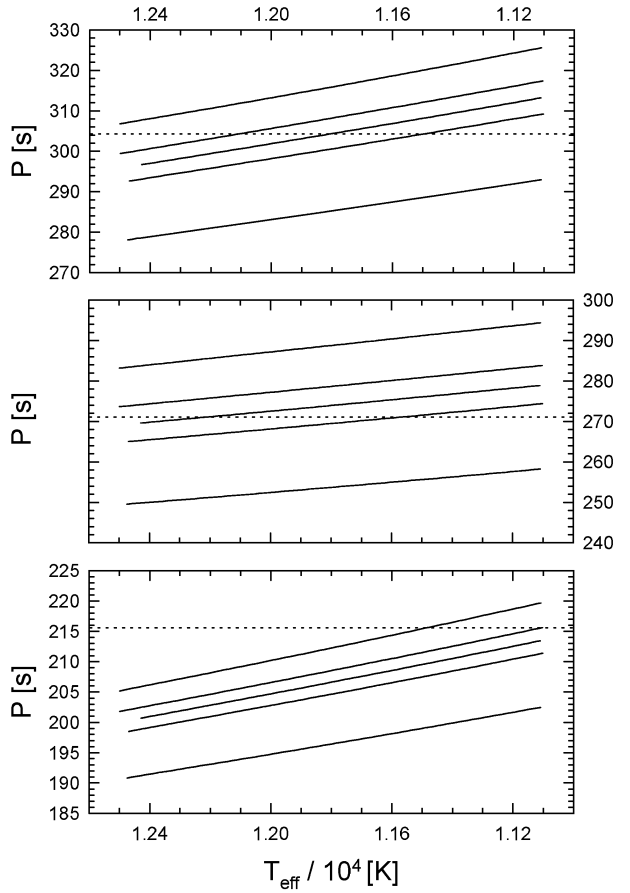

Figure 8. The periods of oscillation for radial order $k = 2$ (lower panel) $k = 3$ (middle panel) and $k = 4$ (upper panel) for the case of models with the thickest hydrogen envelope for each considered value of the stellar mass (from bottom to top, curves correspond to stellar masses of 0.60, 0.55, 0.5375, 0.525 and $0.50\,{\rm M}_\odot$). Notice that the fitting to the observed modes (denoted with horizontal short-dashed lines) should be considered simultaneously at the same effective temperature.

the asteroseismological masses and effective temperatures to be similar to those determined spectroscopically, the best fit is found in the frame of case B for a model with a stellar mass of $0.525\,{\rm M}_\odot$. For the sake of completeness, we also show in Fig. 8 the periods of oscillation for radial order $k = 2, 3, 4$ for the case of models with the thickest hydrogen envelope for each considered value of the stellar mass.

5 DISCUSSION

From the results presented in the preceding section, we conclude that the best fit to observations is found with stellar mass and $M_{\rm H}/M_*$ values that are very sensitive to the adopted mode identification. This has been previously found by Bradley (1998). If we restrict ourselves to analysing the case of G117-B15A only in the frame of asteroseismological models we find no way of being conclusive about the actual values for its stellar mass, $M_{\rm H}/M_*$, and effective temperature. However, if we incorporate the best available spectroscopic determinations of stellar mass and effective temperatures to this analysis and expect them to be compatible with the asteroseismological results, we can discriminate between the identification of modes and also between the other values for the characteristics of the models.

In our opinion, the best fit found in the frame of case A mode identification, though being very good (ψ shows a very deep minimum), occurs however at a temperature and a mass value too low compared to those determined by KA. Nevertheless, for case B we find also a very deep minimum at an effective temperature compatible to that of KA and a mass value almost identical to that given in KA. If we accept that this coincidence is not by chance, we have to conclude that this is the best fit to the structure and evolutionary conditions for G117-B15A. Thus, our results strongly suggest that the hydrogen envelope present in G117-B15A is a thick one with $\log(M_{\rm H}/M_*) \approx -3.8$.

The result regarding the actual amount of hydrogen present in G117-B15A should be considered as a very interesting one. In fact this is a *very thick* hydrogen envelope. Notice that this is the upper limit we considered for this quantity. Because of this fact, in improving our fit we may have considered even thicker envelopes (see Figs 4–6). However, notice that when performing asteroseismology in the way we have, it is not clear if such a procedure would produce stellar models plausible from a physical point of view. Notice that even in the hypothetical case that our artificial procedure for generating the stellar models were able to handle such models, there is no way to be confident that *previous* nuclear burning were effective enough to prevent the existence of such structures in nature. Because of this reason we think it would be a safer procedure to compute the structure of objects of the mass of G117-B15A starting from the hydrogen burning main sequence throughout all the evolutionary stages previous to the WD stage. This is the only way we envisage in order to assess the possible

existence of hydrogen envelopes thicker than those we have considered here. In any case, we should point out that previous detailed calculations of the evolution of objects that finally form a WD predict values near $M_{\rm H} = 10^{-4}\,{\rm M}_{\odot}$ for 0.60-$\rm M_{\odot}$ objects at the start of the cooling branch. However a serious study of this problem would carry us too far afield and is beyond the scope of the present paper.

Finally, we can revisit the estimation of the parallax of G117-B15A as done in Bradley (1998). Because the effective temperature is higher (11 790 versus 11 600 K) and the stellar mass lower (0.52 versus 0.60 $\rm M_{\odot}$, and thus a larger radius), the luminosity ($\log(L/{\rm L}_{\odot}) = -2.44$ versus -2.52) is higher than predicted by Bradley (1998), and for the same visual apparent magnitude $m_V = 11.50$ and bolometric correction of -0.611 mag (Bergeron, Wesemael & Beauchamp 1995a), we find that G117-B15A should have a parallax of 15.89 mas whereas Bradley determined it to be 16.5, which is still too high to be compatible (to a 1σ level) with the determination of 10.5 ± 4.2 mas by van Altena, Lee & Hoffleit (1994). In view of the amount of compatible evidence on the value of the stellar mass and effective temperature, we expect the observed parallax of G117-B15A to be underestimated. It seems difficult to adjudicate the discrepancy in parallax as due to an overestimation of the value of the stellar mass. Were this the case, the value should be significantly lower than found in this work.

6 CONCLUSIONS

In this paper we have studied the structural characteristic of the variable DA white dwarf (WD) G117-B15A by applying the methods of asteroseismology. In doing so we have constructed models of WD evolution considering updated and detailed physical ingredients. It should be remarked than we have included several processes responsible for the diffusion of elements in the WD interior. In particular, we considered gravitational settling and chemical and thermal diffusion. Starting from an artificial model, we have considered several thickness for the outermost hydrogen layer, whereas for the inner helium-, carbon- and oxygen-rich layer we considered the profiles predicted by Salaris et al. (1997). The range of stellar masses we have considered is $0.50 \leq M_*/{\rm M}_{\odot} \leq 0.60$. As far as we are aware, this is the first study in which evolutionary models of DA WDs considering element diffusion have been constructed for a set of values of hydrogen mass fractions.

The evolution of each model sequence were followed down to effective temperature of 12 500 K, from where on we considered the evolution coupled to the oscillations. We considered dipolar, adiabatic g-modes with radial order $k = 1, \ldots, 4$. After constructing the full set of models, we considered the location of the minima of a function conveniently defined for the purpose of fitting. We considered two possible mode identifications according to Bradley (1998): that observed modes 215.2, 271 and 304.4 s correspond to $k = 1, 2, 3$ and $k = 2, 3, 4$. We find that the employment of asteroseismology does not provide an univocal answer about the correct identification of modes present in G117-B15A. However, if we use them together with data deduced from spectroscopy it is found that the identification of the observed periods of 215.2, 271 and 304.4 s with dipolar modes with $k = 2, 3, 4$ is clearly the best.

The results presented in this work (summarized in Table 2) strongly suggest that G117-B15A is a DA WD with a stellar mass of 0.525 $\rm M_{\odot}$, a hydrogen mass fraction $\log(M_{\rm H}/M_*) = -3.83$ and effective temperature $T_{\rm eff} \approx 11800$ K. Notably, the values of the effective temperature and stellar mass are in very nice agreement with those predicted by spectroscopic analysis by Koester & Allard (2000). This represents the main result of the present paper.

Table 2. Summary of results for the best fits.

Quantity	Fitted model	G117-B15A
$M_*/{\rm M}_{\odot}$	0.525	0.53 [(a)]
$\log g$	7.85	7.86 ± 0.14 [(a)]
$T_{\rm eff}$ [K]	11 790	11 900 ± 140[a]
$\log(M_{\rm H}/M_*)$	−3.83	...
$\log(M_{\rm He}/M_*)$	−2.00	...
Parallax [mas]	15.89	10.5 ± 4.2[b]
$P(k = 2)$ [s]	208.80	215.2[c]
$P(k = 3)$	278.85	271.0[c]
$P(k = 4)$	308.57	304.4[c]
$\dot{P}(k = 2)$ [10^{-15} s s^{-1}]	4.43	2.3 ± 1.4[d]
$\dot{P}(k = 3)$	3.22	...
$\dot{P}(k = 4)$	5.76	...

References: [a] Koester & Allard (2000); [b] van Altena et al. (1994); [c] Kepler et al. (1982); [d] Kepler et al. (2000).

While the favoured value for the mass fraction of hydrogen is the maximum we have considered, we have not tried to study the case of slightly (say, 25 per cent) thicker hydrogen layers because our artificial starting evolutionary technique prevents us from being confident with the possibility that such structures can actually be formed in nature. The only way to answer this question is to perform full evolutionary calculations from the initial stages of evolution (hydrogen main sequence) to the WD stage.

Thus, performing full evolutionary studies coupled to oscillations is (in principle) interesting in two senses. First, it can provide significative improvements to the fitting of the oscillatory modes we have found for G117-B15A in this paper. Secondly, and more importantly, it can provide us with a stringent test of the quality of detailed theoretical evolutionary models of WD stars by comparing the theoretical pattern of non-radial oscillations against accurate observational determinations of the oscillatory pattern of WD stars. Work in this sense is in progress and will be published elsewhere.

ACKNOWLEDGMENTS

OGB warmly thanks R. E. Martinez and E. Suarez for their help in getting some key information for the present paper.

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