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Diffusion in helium white dwarf stars

L. G. Althaus★† and O. G. Benvenuto★‡

Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque S/N, 1900 La Plata, Argentina

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ABSTRACT

This paper is aimed at exploring the effects of diffusion on the structure and evolution of low-mass helium white dwarfs. To this end, we solve the multicomponent flow equations describing gravitational settling and chemical and thermal diffusion. The diffusion calculations are coupled to an evolutionary code in order to follow the cooling of low-mass, helium core white dwarf models having envelopes made up of a mixture of hydrogen and helium, as recently suggested by detailed evolutionary calculations for white dwarf progenitors in binary systems. We find that diffusion causes hydrogen to float and the other elements to sink over time-scales shorter than evolutionary time-scales. This produces a noticeable change in the structure of the outer layers, making the star inflate. Thus, in order to compute accurately the mass–radius relation for low-mass helium white dwarfs we need to account for the diffusion processes during (at least) the white dwarf stages of the evolution of these objects. This should be particularly important when studying the general characteristics of binary systems containing a helium white dwarf and a pulsar.

In addition, we present an analytic, approximate model for the outer layers of the white dwarf aimed at interpreting the physical reasons for the change in the surface gravity for low-mass white dwarfs induced by diffusion.

Key words: stars: evolution – stars: interiors – white dwarfs.

1 INTRODUCTION

Both theoretical (Iben & Tutukov 1986; Iben & Livio 1993; Alberts et al. 1996; Sarna, Ergma & Antipova 1999b) and observational (Marsh 1995; Marsh, Dhillon & Duck 1995; Moran, Marsh & Bragaglia 1997; Landsman et al. 1997; Edmonds et al. 1999; Orosz et al. 1999, amongst others) evidence strongly suggests that most low-mass white dwarf (WD) stars would be the result of the evolution of some binary systems. Indeed, mass transfer episodes in close binary systems are required to form helium degenerates with stellar masses less than $\approx 0.5\,\mathrm{M}_\odot$ within a Hubble time (but see Marietta, Burrows & Fryxell 1999). Evolutionary models for these WDs with the emphasis on their mass–radius relations have been the subject of recent investigations, e.g. Althaus & Benvenuto (1997), Benvenuto & Althaus (1998) and Hansen & Phinney (1998).

In a recent study, Sarna et al. (1999b; see also Sarna, Antipova & Ergma 1999a) have presented very detailed calculations of the binary evolution leading to the formation of low-mass helium WDs with stellar masses smaller than $0.25\,\mathrm{M}_\odot$. They found that, after detachment of the Roche lobe, helium cores are surrounded by a massive hydrogen layer of 0.01–$0.06\,\mathrm{M}_\odot$ with a surface hydrogen abundance by mass of $X_\mathrm{H} = 0.35$–0.50. In particular, they found that the minimum mass for the occurrence of a hydrogen flash is dependent on the heavy-element abundances (for example, $0.231\,\mathrm{M}_\odot$ for $Z = 0.003$ and $0.183\,\mathrm{M}_\odot$ for $Z = 0.03$). Such flashes should be critical episodes in determining the thickness of the hydrogen layer atop a very low-mass WD, and thus in establishing the actual time-scale for the further evolution of the star.

Massive hydrogen envelopes for low-mass helium WDs have also been found by Driebe et al. (1998; hereafter DSBH98), who have simulated the binary evolution by forcing a 1-$\mathrm{M}_\odot$ model at the red giant branch to a large mass-loss rate. In particular, DSBH98 (see also Driebe et al. 1999) have followed their calculations down to very low stellar luminosities and derived mass–radius relations in order to analyse recent observational data.

A very important class of objects in which low-mass, helium WDs occur is the binary systems (such as PSR J1012+5307; see e.g. van Kerkwijk, Bergeron & Kulkarni 1996; Sarna, Antipova & Muslimov 1998) composed of a WD and a pulsar. These systems are particularly interesting because we may obtain information about the characteristics of the neutron star by means of studying

★ E-mail: althaus@fcaglp.fcaglp.unlp.edu.ar (LGA); obenvenuto@fcaglp.fcaglp.unlp.edu.ar (OGB)

† Member of the Carrera del Investigador Científico y Tecnológico, Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina.

‡ Member of the Carrera del Investigador Científico, Comisión de Investigaciones Científicas de la Provincia de Buenos Aires, Argentina.

the helium WD component of the pair. Thus, by studying such binaries we can, in principle, constrain the mass of the neutron star (and consequently the still quite uncertain properties of the cold – nuclear – matter equation of state) and its age (with direct implications on the evolution of the magnetic field, if present). In order to obtain firm conclusions about the general characteristics of these objects, it is fundamental to have available helium WD models as physically sound as possible.

It is worth remarking that DSBH98 neglect the effect of diffusion in their calculations even during the WD stages. Diffusion is expected to cause all heavier elements than hydrogen to sink below the outer layers, thereby leading to changes in the surface gravity of their models (see Appendix A for a simple physical explanation of the origin of such changes). These changes are expected to be particularly noticeable in the case of low-mass WDs, because the hydrogen layer embraces a large fraction of the total mass (and a much larger fraction of the stellar radius), as predicted by the evolutionary calculations cited above. It is the main goal of the present paper to show that the modifications to the structure of the outer layers of these low-mass, helium WDs are not negligible and must be taken into account when assessing stellar masses from mass–radius relations for these objects.

The study of diffusion processes in WDs is a subject that has received considerable attention since the early recognition by Schatzman (1958) on the fundamental role played by diffusion in the evolution of the superficial chemical composition of WDs. Indeed, over the last three decades, numerous studies, based on an increasing degree of sophistication both in theoretical and observational techniques, have convincingly demonstrated that gravitationally induced diffusion is an extremely efficient process in separating elements in the envelopes of WDs. These studies have shown that diffusion processes cause elements heavier than the main atmospheric constituents to sink below the photosphere over time-scales much smaller than the evolutionary time-scales, thus providing an explanation for the purity of almost all WD atmospheres (Fontaine & Michaud 1979; Alcock & Illarionov 1980b; Muchmore 1984; Iben & MacDonald 1985; Paquette et al. 1986b; Dupuis et al. 1992, amongst others). In addition, sophisticated models invoking an interplay among various mechanisms such as diffusion, convection, accretion and wind mass loss have been put forward to account for the presence of heavy elements in very small proportions detected in the spectrum of some WDs (see e.g. Fontaine & Michaud 1979; Vauclair, Vauclair & Greenstein 1979; Pelletier et al. 1986; Paquette et al. 1986b; Iben & MacDonald 1985; Dupuis et al. 1992; MacDonald, Hernanz & José 1998).

Interestingly enough, diffusion in low-mass WDs has been marginally addressed in some of the works mentioned above. In particular, Alcock & Illarionov (1980a) have found diffusion time-scales for a WD model with a mass as low as $0.2\,M_\odot$ that are comparable to the evolutionary time-scales. In fact, diffusion times appear to increase significantly as stellar mass is decreased (see Muchmore 1984). On the other hand, Paquette et al. (1986b) have found, on the basis of improved diffusion coefficients, that the diffusion time-scales remain smaller than the cooling ages.

In this paper, we investigate the effects of diffusion on models of low-mass helium WDs with envelopes consisting partly of hydrogen and partly of helium, as predicted by recent evolutionary calculations of WD progenitors in close binary systems. The diffusion calculations are based on the multicomponent treatment of the gas developed by Burgers (1969) appropriate for the case we want to study here, thus avoiding the trace element approximation invoked in most of diffusion studies. Gravitational settling, thermal and chemical diffusion are included in our calculations and the relative importance of thermal diffusion with respect to gravitational settling in these models is discussed. In order to calculate self-consistently the dependence of the structure of our WD models on the varying abundances in the envelope, the diffusion calculation has been coupled with our evolutionary code.

The present paper is organized as follows. In Section 2 we briefly describe our evolutionary code and initial models. Section 3 is devoted to presenting the diffusion equations and the method we follow to solve them. In Section 4 we detail our main results and, finally, Section 5 is devoted to making some concluding remarks.

2 THE EVOLUTIONARY CODE AND INITIAL MODELS

The numerical code we employ to simulate the evolution of our helium WD models has been detailed in our previous works on WDs and we refer the reader to Althaus & Benvenuto (1997, 1998) for details about both the physical ingredients we incorporated and the procedure we followed to generate the initial models. In particular, the equation of state for the low-density regime is that of Saumon, Chabrier & Van Horn (1995) for hydrogen and helium plasmas, and the treatment for the high-density regime (solid and liquid phases) includes ionic contributions, Coulomb interactions, partially degenerate electrons, electron exchange and Thomas–Fermi contributions at finite temperature. High-density conductive opacities and the various mechanisms of neutrino emission are taken from the works of Itoh and collaborators (see Althaus & Benvenuto 1997 for details). Radiative opacities for high temperatures are those of OPAL (Iglesias & Rogers 1993), whilst for the low-temperature regime we employ the Alexander & Ferguson (1994) molecular opacities. With respect to the energy transport by convection, we adopt the full-spectrum turbulence theory developed by Canuto, Goldman & Mazzitelli (1996), which represents an improvement over the mixing length theory usually employed in WD studies.

In this work we have not computed the binary evolution leading to the formation of low-mass WDs (see Section 1). Rather, we have constructed our initial models by using an artificial evolutionary procedure described by Benvenuto & Althaus (1998). Specifically, the initial models have been constructed from a helium WD model, to which we have added an artificial energy release up to the moment in which the model is very luminous. Then, we switch it off smoothly, obtaining a starting model very close to the cooling branch. After such 'heating', models experience a transitory relaxation to the desired WD structure from which our cooling sequences start, and which is considered as meaningful in this paper. The main shortcoming of this procedure is that it does not predict the amount of hydrogen left in the WD as a relic of its previous evolution. This quantity must be fixed by employing other arguments, e.g. by taking the values found in full evolutionary calculations. For a quantitative justification of this method, we refer the reader to Appendix B.

3 DIFFUSION EQUATIONS AND METHOD OF CALCULATION

To treat the problem of diffusion in helium WDs, we have developed a code that solves the equations describing gravitational settling and chemical and thermal diffusion. Radiative levitation, which is important for determining photospheric composition of hot WDs (Fontaine & Michaud 1979), has been neglected. This assumption is justified because we are interested in the chemical evolution occurring quite deep in the star. In order to avoid the trace element approximation usually adopted in most studies of diffusion in WDs, we have performed our diffusion calculations following the method presented by Burgers (1969) for treating multicomponent gases, which is based on the use of velocity moments of the Boltzmann equation. In the context of WD evolution, this method, which is appropriate for the case we want to study here, has been used by Muchmore (1984) and Iben & MacDonald (1985). Recently, it has been applied by MacDonald et al. (1998) to address the problem of carbon dredge-up in carbon–oxygen WDs with helium-rich envelopes.

Under the influence of gravity, partial pressure, thermal gradients and induced electric fields (we neglect stellar rotation and magnetic fields) the diffusion velocities in a multicomponent plasma satisfy the rather complicated set of diffusion equations ($N-1$ independent linear equations, Burgers 1969)

$$\frac{\mathrm{d}p_i}{\mathrm{d}r} - \frac{\rho_i}{\rho}\frac{\mathrm{d}p}{\mathrm{d}r} - n_i Z_i eE = \sum_{j\neq i}^{N} K_{ij}(w_j - w_i) + \sum_{j\neq i}^{N} K_{ij} z_{ij} \frac{m_j r_i - m_i r_j}{m_i + m_j}, \tag{1}$$

and heat flow equation (N equations)

$$\frac{5}{2} n_i k_\mathrm{B} \nabla T = -\frac{5}{2}\sum_{j\neq i}^{N} K_{ij} z_{ij} \frac{m_j}{m_i + m_j}(w_j - w_i) - \frac{2}{5} K_{ii} z''_{ii} r_i - \sum_{j\neq i}^{N} \frac{K_{ij}}{(m_i + m_j)^2}(3m_i^2 + m_j^2 z'_{ij} + 0.8 m_i m_j z''_{ij}) r_i + \sum_{j\neq i}^{N} \frac{K_{ij} m_i m_j}{(m_i + m_j)^2}(3 + z'_{ij} - 0.8 z''_{ij}) r_j. \tag{2}$$

In the above equations, p_i, ρ_i, n_i, Z_i and m_i denote, respectively, the partial pressure, mass density, number density, mean charge and mass for species i (N means the number of ionic species plus electron). T, k_B and ∇T are the temperature, Boltzmann constant and temperature gradient, respectively. The unknown variables are the diffusion velocities with respect to the centre of mass, w_i, and the residual heat flows r_i (for ions and electrons). In addition the electric field E has to be determined. The resistance coefficients (K_{ij}, z_{ij}, z'_{ij} and z''_{ij}) are from Paquette et al. (1986a), and average ionic charges are treated following an approximate pressure ionization model as given by Paquette et al. (1986b), which is sufficient for our purposes.

Thus, we have $2N-1$ equations with $2N+1$ unknowns. To complete the set of equations, we use the conditions for no net mass flow with respect to the centre of mass,

$$\sum_i A_i n_i w_i = 0, \tag{3}$$

and no electrical current,

$$\sum_i Z_i n_i w_i = 0. \tag{4}$$

In terms of the gradient in the number density, we can transform equation (1) to

$$\frac{1}{n_i}\left[\sum_{j\neq i}^{N} K_{ij}(w_i - w_j) + \sum_{j\neq i}^{N} K_{ij} z_{ij} \frac{m_i r_j - m_j r_i}{m_i + m_j}\right] - Z_i eE = \alpha_i - k_\mathrm{B} T \frac{\mathrm{d}\ln n_i}{\mathrm{d}r}, \tag{5}$$

where

$$\alpha_i = -A_i m_\mathrm{H} g - k_\mathrm{B} T \frac{\mathrm{d}\ln T}{\mathrm{d}r}, \tag{6}$$

and A_i, m_H, g and T are the atomic mass number, hydrogen atom mass, gravity and temperature, respectively. Let us write the unknowns w_i, r_i and E in terms of the gradient of ion densities in the form (similarly for r_i and E)

$$w_i = w_i^\mathrm{gt} - \sum_{\mathrm{ions}(j)} \sigma_{ij} \frac{\mathrm{d}\ln n_j}{\mathrm{d}r}, \tag{7}$$

where w_i^gt stands for the velocity component resulting from gravitational settling and thermal diffusion. The summation in equation (7) is to be made for the ions only. With equations (2) and (5) together with (3) and (4) we can easily find the components w_i^gt and σ_{ij} by matrix inversions (LU decomposition).

Having arrived at this point, we can now solve the continuity equation to find the evolution of element distribution throughout the star. To this end, we follow the treatment given by Iben & MacDonald (1985) and we write the continuity equation as

$$\frac{\partial n_i}{\partial t} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(w_i^\mathrm{gt} n_i - \sum_j \frac{n_i}{n_j}\sigma_{ij}\frac{\partial n_j}{\partial r}\right)\right], \tag{8}$$

which is solved by means of a semi-implicit, finite difference scheme. In particular, we follow the evolution of the isotopes ^{1}H, ^{3}He, ^{4}He, ^{12}C and ^{16}O. The boundary conditions for equation (8) are $\partial n_i/\partial r = 0$ at the stellar centre and $n_i = 0$ at the surface. In order to calculate the dependence of the structure of our WD models on the varying abundances self-consistently, the set of equations describing diffusion has been coupled to our evolutionary code. To this end, we replace the outer boundary condition by the continuity of the abundance by mass fraction X_i in the envelope. At the centre, we perform a second-order expansion for n_i. We want to mention that after computing the change of abundances by effect of diffusion, they are evolved according to nuclear reactions and convective mixing.

4 RESULTS

In this section we shall describe the main results of our calculations. Using the evolutionary code and the treatment for diffusion described in the preceding section, we have computed the evolution of low-mass helium WD models with envelopes consisting partly of hydrogen and partly of helium, as predicted by recent evolutionary calculations of WD progenitors in close binary systems. Here we shall mainly be concerned with the evolution of the distribution of element abundances below the stellar surface and its consequences for the WD evolution and mass–radius relation. Specifically, we have considered WD models with stellar masses of 0.414, 0.30 and 0.195 $\mathrm{M}_\odot$. The initial chemical profiles we adopted for these models are the following: for the 0.414-$\mathrm{M}_\odot$ models we used an envelope characterized by a mass fraction of $M_\mathrm{env}/M_* = 1\times10^{-3}$ (where M_env is the mass of the envelope where hydrogen is present and M_* is the stellar mass) and a hydrogen abundance by mass of $X_\mathrm{H} = 0.7$; for the 0.3-$\mathrm{M}_\odot$ models we considered $M_\mathrm{env}/M_* = 2\times10^{-3}$ and $X_\mathrm{H} = 0.7$ and finally, for the 0.195-$\mathrm{M}_\odot$ models we used $M_\mathrm{env}/M_* = 6\times10^{-3}$ with $X_\mathrm{H} = 0.52$. These envelope mass and hydrogen abundance values correspond approximately to those found by DSBH98. It is worth

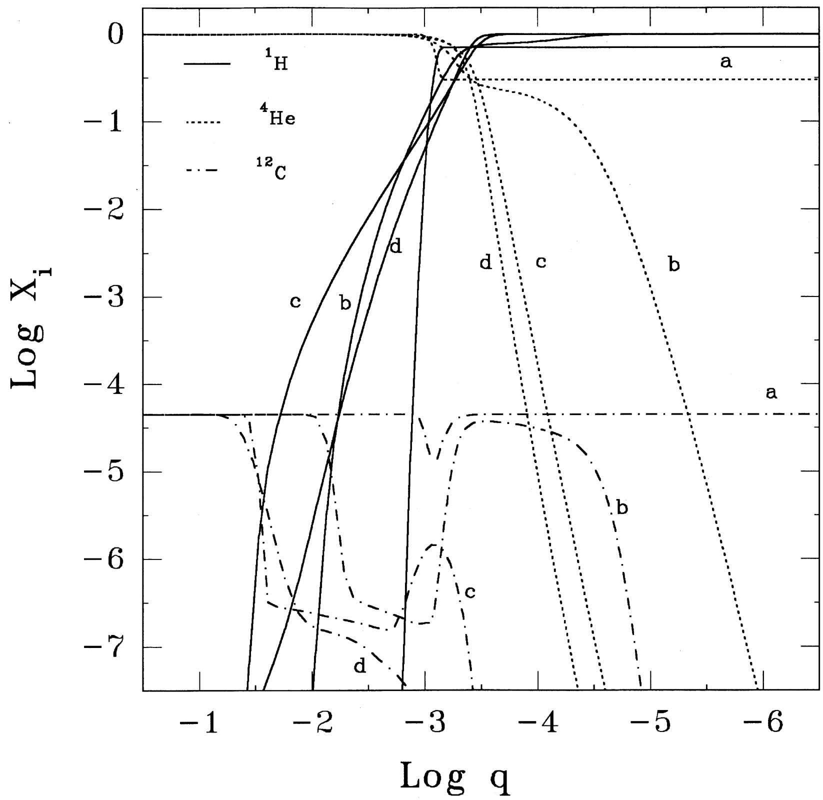

Figure 1. Abundance by mass of ^{1}H, ^{4}He and ^{12}C versus the outer mass fraction $q = 1 - M_r/M_*$ for 0.414-$M_\odot$ WD models at different evolutionary stages characterized by $\log(T_{\rm eff}) = 4.411$, 4.362, 4.170 and 3.740 (denoted by letters 'a', 'b', 'c' and 'd', respectively). Diffusion processes are allowed to operate from model 'a' on. Note that diffusion substantially alters the initial chemical profiles, rapidly leading to pure hydrogen envelopes. Note also the tail of hydrogen diffusing downwards thorough hotter and ^{12}C-rich helium layers.

mentioning that our 0.195-$M_\odot$ initial model has an outer convection zone embracing an outer mass fraction of $M_{\rm conv}/M_* \approx 1 \times 10^{-10}$. This convection zone, which is a result of the high helium content of the envelope ($X_{\rm He} \approx 0.5$), rapidly disappears as a result of diffusion. Indeed, as diffusion time-scales at the bottom of this convection zone are extremely short, the helium abundance is rapidly depleted from the outermost layers. This leads to a reduction of the opacities throughout these layers, forcing convection to retreat towards the surface regions.

With regard to the election of the mass values for the envelopes of our models, some words are in order. In particular, the envelope masses have been finely adjusted in order for hydrogen-burning reactions to represent a minor contribution to surface luminosity of the model. For somewhat thicker envelopes than adopted here, we find that hydrogen burning increases substantially, thus causing the WD evolution to be considerably slower. This being the case, the separation between hydrogen and helium deep in the star resulting from diffusion processes would take place at larger effective temperature (hereafter $T_{\rm eff}$) values than quoted in this work. We also want to stress here that a weak point of our study is that, because of the artificial procedure we employed to generate the initial models, diffusion was allowed to operate once the model has reached its cooling branch at intermediate $T_{\rm eff}$ values (see Section 2). In view of these considerations, the $T_{\rm eff}$ values at which element separation occurs in our models should be considered as lower limits.

We begin by examining Figs 1 and 2 in which the profile of ^{1}H, ^{4}He and ^{12}C abundances for 0.414 and 0.195-$M_\odot$ helium WD models is shown for some selected $T_{\rm eff}$ values. Starting with models with initially homogeneous envelopes, note that diffusion rapidly leads to pure hydrogen envelopes, as one could expect from previous diffusion studies in WDs. Note that, although to a lesser extent, the same is true even for models with stellar masses as low as $\approx 0.2\,M_\odot$. A feature of interest shown by these figures is the existence of a tail of hydrogen penetrating downwards through hotter helium layers as a result of chemical diffusion (see Iben & MacDonald 1985 for a similar finding in the case of carbon–oxygen WDs of intermediate masses). As cooling proceeds, electron degeneracy reaches the bottom of the hydrogen layers and chemical diffusion begins thus to play a minor role there. From then on, gravitational settling and thermal diffusion take over, thus forcing the tail of hydrogen distribution to move upwards. This last trend is not found in the 0.195-$M_\odot$ models (at least for the range of $T_{\rm eff}$ values we explored) because of the lower electron degeneracy that characterized them. It is worthwhile to mention that, as stellar luminosity decreases, the thickness of the hydrogen convection zone begins to increase appreciably as a consequence of the sink of the region of partial ionization. Thereby, convective mixing could in principle dredge up some carbon or heavier elements that were falling down. However, diffusion time-scales for carbon are so short that it remains out of reach of the convection zone of the models analysed here. Thus, carbon cannot be brought back up to the surface region from the deep envelope by mixing processes as the star cools. This result is likewise true even for the 0.195-$M_\odot$ model, the convection zone of which

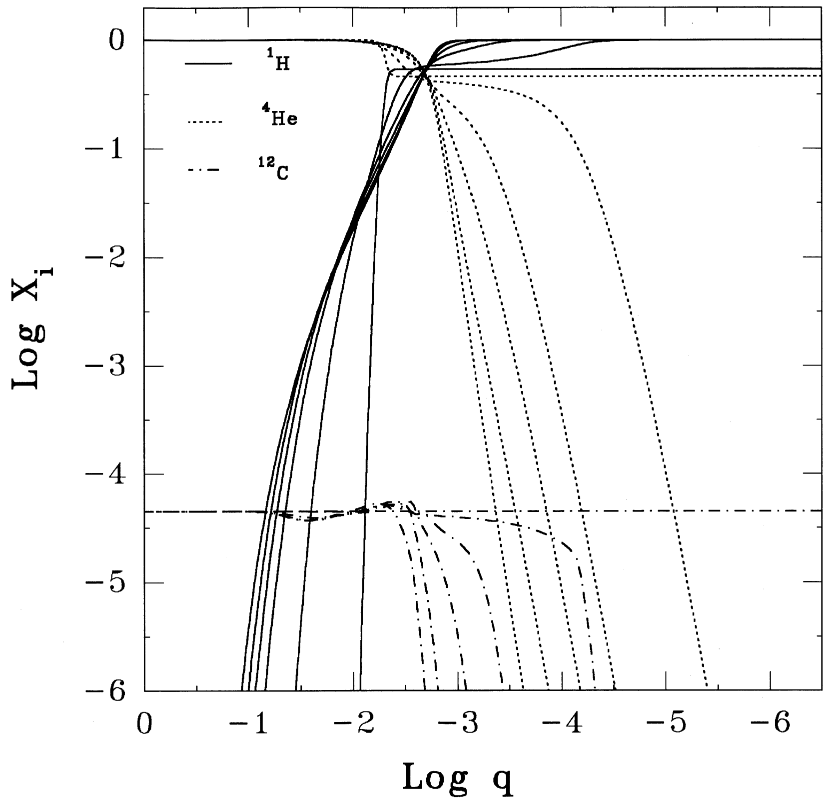

Figure 2. Abundance by mass of ^{1}H, ^{4}He and ^{12}C versus the outer mass fraction $q = 1 - M_r/M_*$ for 0.195-M$_\odot$ WD models at different evolutionary stages. Starting with models with an initially homogeneous envelope with $X_{\rm H} = 0.538$ at $\log(T_{\rm eff}) = 4.10$ (from where diffusion is allowed to operate), the following models are characterized by $\log(T_{\rm eff}) = 4.02$, 3.93, 3.88, 3.84 and 3.762 [at low abundances and for each set of curves, the smaller the q the higher $\log(T_{\rm eff})$]. Note that diffusion substantially alters the initial chemical profiles, rapidly leading to pure hydrogen envelopes. Note also the tail of hydrogen diffusing downwards thorough hotter helium layers.

reaches down to $M_{\rm conv}/M_* \approx 10^{-4.5}$ at the smallest $T_{\rm eff}$ values considered here (5700 K).[1]

Total diffusion velocities for hydrogen and helium throughout the interior of our 0.414- and 0.195-M$_\odot$ models are illustrated, respectively, in Figs 3 and 4 at two selected $T_{\rm eff}$ values. A negative velocity means that a given element sinks into the star. The first observation we can make from the results corresponding to the 0.414-M$_\odot$ model is that the diffusion velocities are much smaller at lower $T_{\rm eff}$ values. It is also clear that at high $T_{\rm eff}$ values (curves 1), the tail of the hydrogen distribution diffuses downwards very rapidly as a result of chemical diffusion. This trend continues until the $T_{\rm eff}$ have been lowered down to $\log T_{\rm eff} \approx 4.07$. With further cooling, the tail of hydrogen begins to diffuse upwards (as it is reflected by the change of sign of the hydrogen velocity curve). By contrast, the chemical diffusion of hydrogen through hotter helium layers in the 0.195-M$_\odot$ model (Fig. 4) persists over the entire cooling process computed here. As mentioned earlier, this is because conditions for electron degeneracy at the bottom of the hydrogen envelope are reached at much lower $T_{\rm eff}$ values in less massive models.

The diffusion velocities can be translated into diffusion time-scales $\tau_{\rm D}$ as

$$\tau_{\rm D}^{-1} = V_i \frac{{\rm d} \ln X_i}{{\rm d}r}, \tag{9}$$

[1] We should note, however, that at $T_{\rm eff} \approx 4700$ K, the convection zone of the 0.195-M$_\odot$ model reaches a maximum depth of $M_{\rm conv}/M_* \approx 10^{-2.7}$. Thus, such a statement should be taken with caution at such low $T_{\rm eff}$ values.

where V_i and X_i are, respectively, the total diffusion velocity and the abundance by mass of species i. In Figs 5 and 6 we plot the diffusion time-scales of hydrogen and helium as function of depth for the same stellar masses and $T_{\rm eff}$ values as those referred to as in Figs 3 and 4. In the case of the 0.414-M$_\odot$ model, it is immediately evident that, even quite deep in the star, helium diffuses from the outer layers over time-scales much shorter than evolutionary time-scales, defined as

$$\tau_{\rm ev} = T_{\rm C}/\dot{T}_{\rm C}, \tag{10}$$

where $T_{\rm C}$ is the central temperature of the model. However, for the 0.195-M$_\odot$ model, helium is depleted from the top of the hydrogen–helium envelope at a rate comparable to the rate of cooling. In the $\log g$–$\log T_{\rm eff}$ plane this is translated, compared with the case of the 0.414-M$_\odot$ model, into a much slower convergence between the track calculated under the assumption of diffusion and the track obtained assuming a pure hydrogen envelope (see below). Also clearly noticeable from Fig. 5 and 6 are the much shorter diffusion time-scales of hydrogen at the tail of its distribution, compared with evolutionary time-scales. This is particularly true for models at high $T_{\rm eff}$ values.

We have also analysed the role played by thermal diffusion in the element separation processes acting in low-mass WDs. For this purpose, we show in Figs 7 and 8 the relative importance of thermal diffusion with respect to gravitational settling as function of depth for 0.414- and 0.195-M$_\odot$ models at three different $T_{\rm eff}$ values (in these figures, $V_{\rm TH+G}$ and $V_{\rm G}$ are, respectively, diffusion velocities for hydrogen and helium composition calculated with

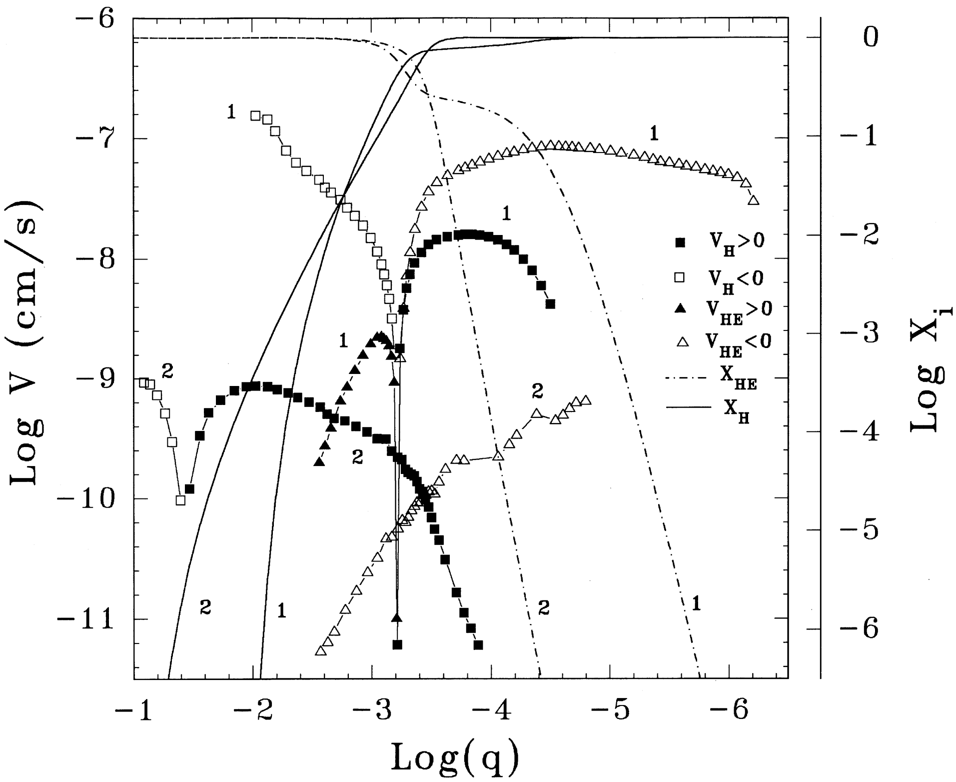

Figure 3. Diffusion velocities for hydrogen (squares) and helium (triangles) elements in terms of the outer mass fraction q. The results correspond to 0.414-$M_{\odot}$ helium WD models at $\log(T_{eff}) = 4.36$ (1) and 4.0 (2). Hollow symbols means that a given element sinks into the star (negative velocity), filled symbols that the element diffuses upwards (positive velocity). The abundance by mass (X_i) of hydrogen and helium at the same $\log(T_{eff})$ values is also shown. Note the much smaller diffusion velocities in the model with lower T_{eff}. Note also that at high $\log(T_{eff})$ values, chemical diffusion causes some hydrogen to have high negative velocities, a trend that is reversed at somewhat lower $\log(T_{eff})$ values.

and without thermal diffusion). We find that gravitational settling always dominates over thermal diffusion (see also Paquette et al. 1986b for a similar result in the context of more massive WDs). However, thermal diffusion is not entirely negligible. Indeed, at high T_{eff} values, we find that for hydrogen diffusing in a helium-rich plasma, the diffusion velocity resulting from gravitational settling alone may be increased up to 60 per cent when thermal diffusion is included. As thermal diffusion acts in the same direction as gravitational settling, the neglect of thermal diffusion would imply a much higher rate at which the tail of hydrogen sinks downwards. For helium diffusing in hydrogen-rich envelopes, note that at high T_{eff}, thermal diffusion contributes about 30 per cent of the gravitational settling velocity. In this case, the neglect of thermal diffusion would lead to an underestimate of the rate at which helium diffuses downwards.

It is well known that as a WD cools down, an outer convection zone begins to develop as result of partial ionization of the main atmospheric constituents (hydrogen in our case). In our models with pure hydrogen envelopes this takes place at $T_{eff} \approx 16\,000$ K (Benvenuto & Althaus 1998). As evolution proceeds, the convection zone digs deep into the star until it reaches the domain of degeneracy. Any element i other than hydrogen accreted by the star will be depleted from the base of the convection zone over a time-scale given by (Muchmore 1984)

$$\theta = \frac{g}{4\pi G}\frac{q_{conv}}{\rho V_i}, \tag{11}$$

where $q_{conv} = 1 - (M_r)_{conv}/M_*$ is the outer mass fraction at the bottom of the convection zone [$(M_r)_{conv}$ denotes the mass coordinate at such a place], g and V_i are, respectively, the gravity and the diffusion velocity of species i with respect to the background of hydrogen at this point, G is the gravitational constant and ρ the density. The diffusion time-scales of helium, carbon and magnesium at the base of the hydrogen-rich convection zone are shown in Fig. 9 as a function of T_{eff}. The results correspond to helium WD models with stellar masses of 0.415, 0.3 and 0.195 $M_{\odot}$ (curves labelled as 1, 2 and 3, respectively). Needless to say, such time-scales depend on the depth reached by the convection zone, which is affected by the uncertainties inherent in the convection model employed. In this sense, our calculations were made in the framework of the Canuto et al. (1996) theory of convection, which represents an important improvement over the mixing- length theory employed in most of WD studies. Note that over almost the entire T_{eff} range, the depletion of the abundance of elements from the convection zone occurs over time-scales much smaller than the evolutionary time-scales ($\tau_D \ll \tau_{ev}$). For less massive models, however, diffusion time-scales become comparable to evolutionary time-scales at very low T_{eff} values. Indeed, the convection zone penetrates much deeper in a less massive WD ($\log q_{conv} = -2.7$ versus -4.5 for 0.195- and 0.4-$M_{\odot}$ models), and this is primarily responsible for longer diffusion time-scales. Coupled with the fact that these WDs evolve more rapidly, it is not surprising that, at advanced stages of evolution, the diffusion time-scale becomes comparable to the evolutionary one. Thus, metals accreted from interstellar clouds could be maintained in the atmosphere of these cool, low-mass

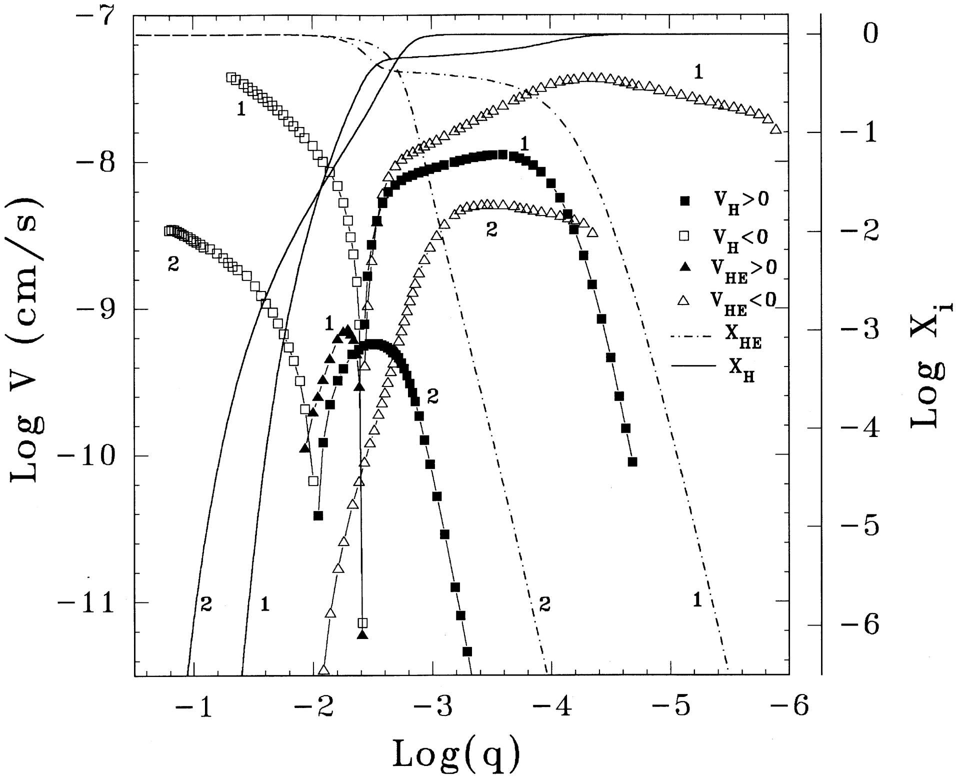

Figure 4. Same as Fig. 3 but for 0.195-M$_\odot$ helium WD models. Here, we choose two evolutionary stages corresponding to $\log(T_{\rm eff}) = 4.02$ (1) and 3.84 (2). Note that, in contrast with the behaviour found for the 0.414-M$_\odot$ model, chemical diffusion causes some hydrogen to have negative velocities even at very low $\log(T_{\rm eff})$ values.

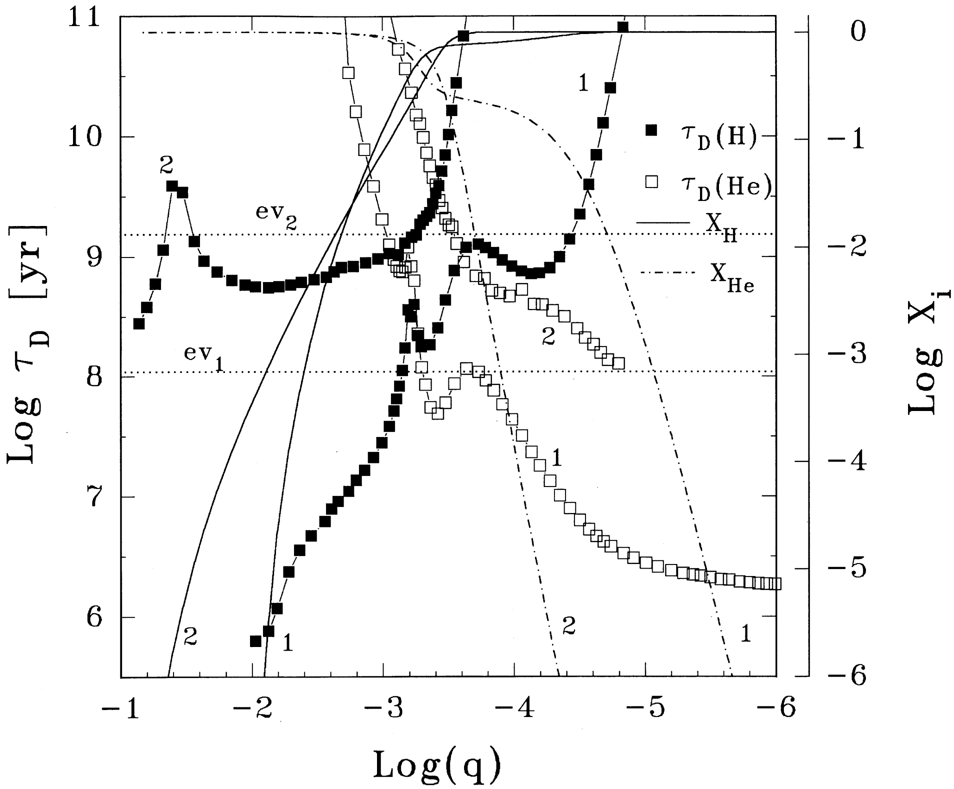

Figure 5. The diffusion time-scale for hydrogen and helium (filled and hollow squares, respectively) elements in terms of the outer mass fraction q. The results correspond to 0.414-M$_\odot$ helium WD models at the same $\log(T_{\rm eff})$ values as those of Fig. 3 (i.e. 4.36 and 4.0, denoted by '1' and '2' respectively). Also shown are the evolutionary time-scales (dotted lines) and the abundance by mass (X_i) of hydrogen and helium at the same $\log(T_{\rm eff})$ values. Note that helium diffuses from the outer layers over time-scales much shorter than the evolutionary time-scales.

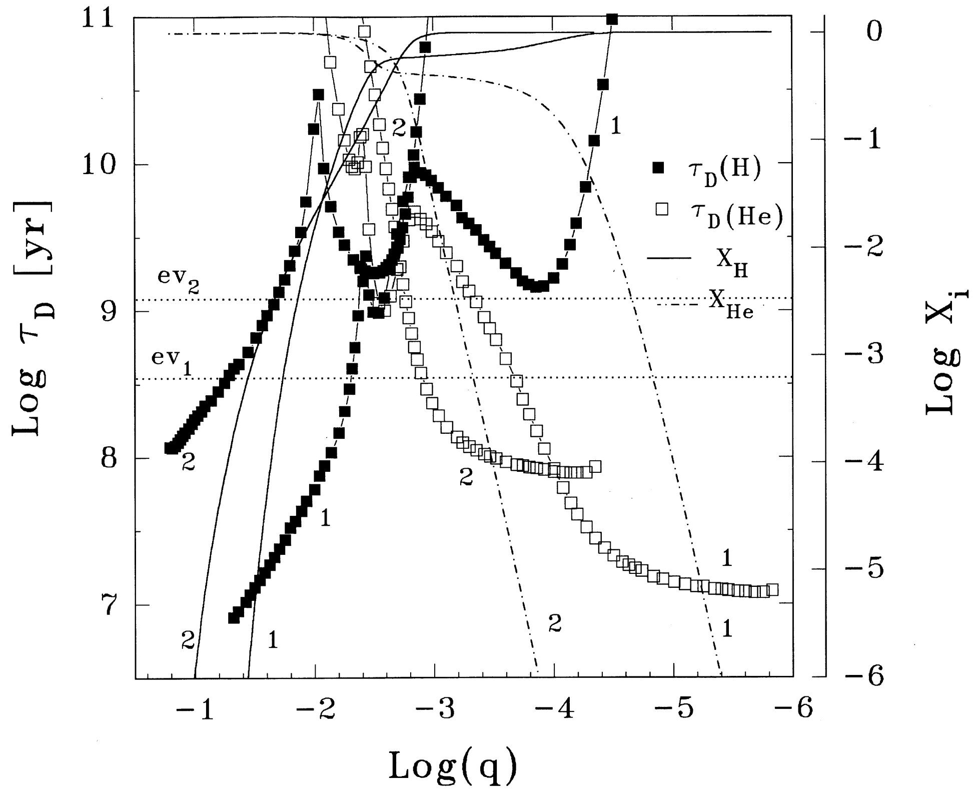

Figure 6. Same as Fig. 5 but for 0.195-$M_{\odot}$ helium WD models. Here, we choose two evolutionary stages corresponding to $\log(T_{\rm eff}) = 4.02$ (1) and 3.84 (2) (see also Fig. 4). Note that unlike the behaviour encountered in more massive models, helium is depleted from the top of the hydrogen–helium envelope over time-scales comparable to evolutionary time-scales.

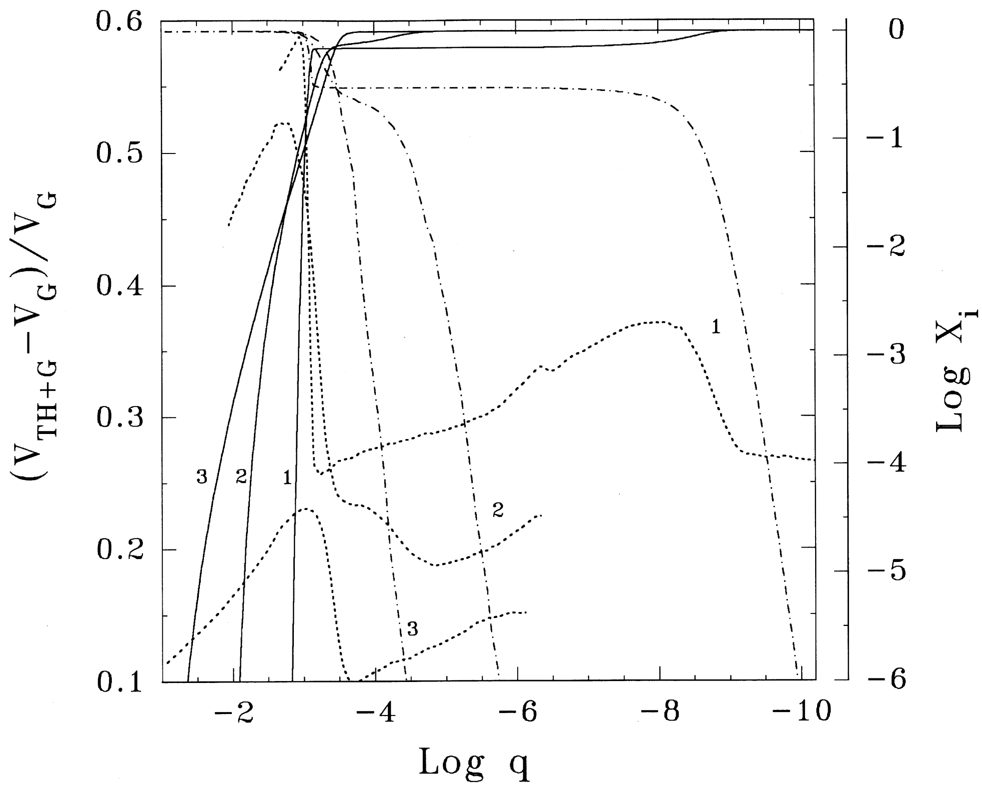

Figure 7. Importance of thermal diffusion for hydrogen and helium composition with respect to gravitational settling in terms of the outer mass fraction q. The results, represented by dotted lines, correspond to 0.414-$M_{\odot}$ helium WD models at $\log(T_{\rm eff}) = 4.41$ (1), 4.36 (2) and 4.0 (3). The abundance by mass (X_i) of hydrogen (solid lines) and helium (dot–dashed lines) at the same $\log(T_{\rm eff})$ values is also shown. Note that, at high $T_{\rm eff}$, thermal diffusion plays a non-negligible role, particularly in those regions where helium is a main constituent and hydrogen is a trace element.

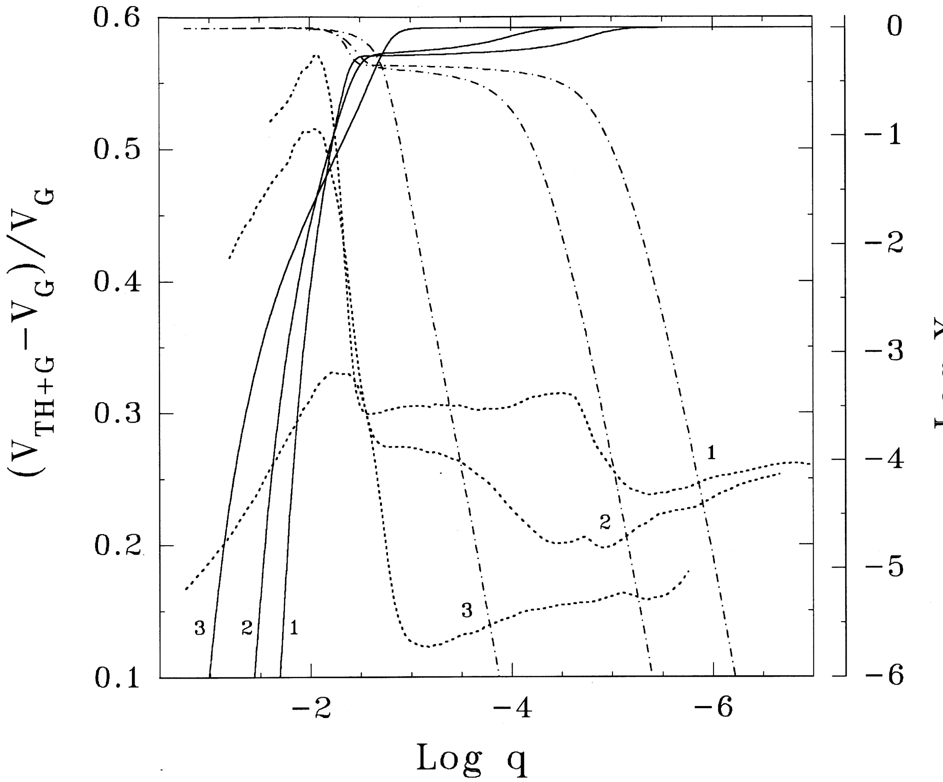

Figure 8. Same as Fig. 7 but for 0.195-$M_\odot$ helium WD models. Here, the evolutionary stages correspond to $\log(T_{eff}) = 4.063$ (1), 4.018 (2) and 3.836 (3). Note that thermal diffusion is particularly important in those regions where hydrogen is a trace element.

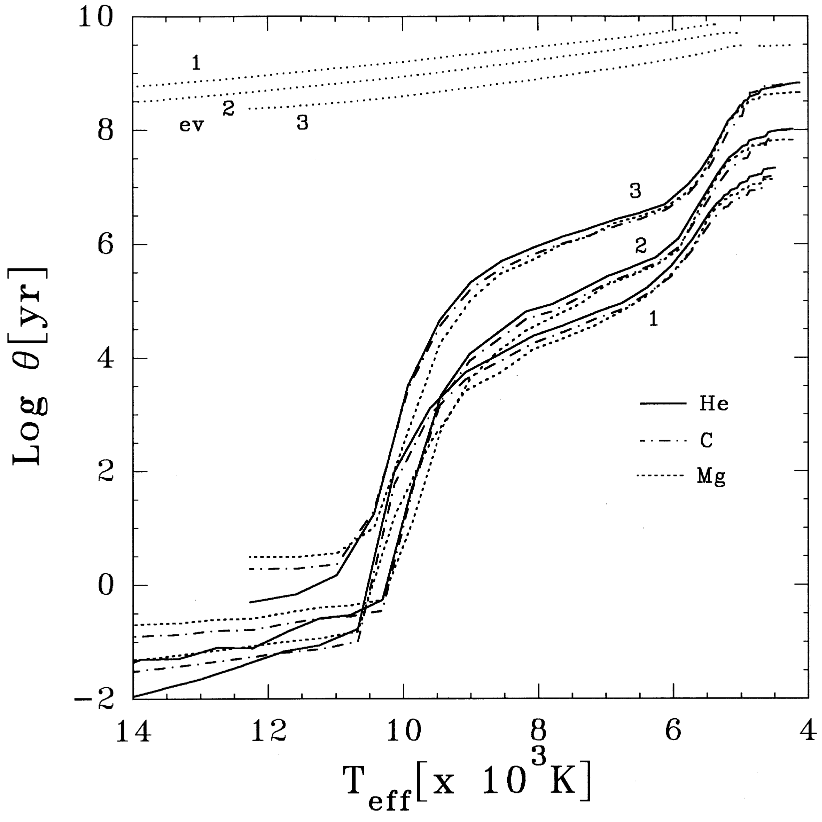

Figure 9. Diffusion time-scale (defined by equation 11) of helium, carbon and magnesium evaluated at the base of the pure-hydrogen convection zone in terms of the T_{eff}. The results correspond to helium WD models with stellar masses of 0.415, 0.3 and 0.195 $M_\odot$ (curves labelled as '1', '2' and '3', respectively). Note that, except for very low $\log(T_{eff})$ values, the depletion of the abundance of elements heavier than hydrogen from the convection zone occurs over time-scales much smaller than the evolutionary time-scales (dotted lines) defined by equation (10).

WD stars for a long time, thus favouring their detection. More specifically, it is apparent from the figure that the diffusion time-scales at the base of the convection zone of the 0.195-$M_\odot$ model become as high as 3×10^8 yr as $T_{\rm eff}$ is lowered below $\approx$5000 K. This time-scale is larger than the typical time interval between encounters with interstellar clouds (10^7–10^8 yr; Wesemael 1979). Also, in the case of 0.3-$M_\odot$ models, diffusion time-scales may become comparable to 10^8 yr. Thus, the presence of metals in the spectra of such low-mass WDs does not necessarily reflect a recent encounter with clouds. Another feature shown by Fig. 9 worthy of comment is that, in general, diffusion time-scales of various elements do not differ by large amounts. It is also worth noting that diffusion time-scales at the base of the convection zone increase monotonically with decreasing $T_{\rm eff}$, as observed in Fig. 9 (see also Paquette et al. 1986b). Another observation we can make from this figure is that at high $T_{\rm eff}$ values, when models are characterized by shallow convection zones, helium ionization is not complete, making this element have shorter diffusion time-scales than the others (see Paquette et al. 1986b for a similar behaviour).

Now, let us discuss the effects of diffusion induced on the structure of the WDs. This is an important point, because, as it will be shown below, diffusion modifies the mass–radius relation (and thus the surface gravities) substantially.

If we compute the structure of a WD of a given stellar mass and hydrogen envelope mass, it is clear that models with abundance profiles such as those of DSBH98 (a helium core surrounded by an envelope made up of a mixture of hydrogen and helium) must have gravities larger than those corresponding to models in which hydrogen and helium are not mixed at all (see Appendix A). Models with a hydrogen outer layer fully separated from helium have been computed by Benvenuto & Althaus (1998) under the extreme hypothesis that diffusion was so efficient that it was possible to separate hydrogen from helium in the evolutionary stages previous to those computed there.

Perhaps the simplest way to gauge the actual importance of this effect on the mass–radius relation is to compute the evolution of a WD model with the outermost layer fully mixed but allowing diffusion to operate. In a case in which diffusion were irrelevant, we would find results very similar to those of DSBH98. In contrast, if diffusion were important we would obtain surface gravities approaching asymptotically the results predicted by models with hydrogen and helium fully separated.

To place this assertion on a more quantitative basis, we show in Fig. 10 the surface gravity g values versus $T_{\rm eff}$ for our models. Note that for less massive models the change in surface gravities brought about by diffusion is appreciable. Note also that, as cooling proceeds, the predicted surface gravities resemble those calculated under the assumption of complete separation between the hydrogen and helium (with the same initial amount of hydrogen), as expected (see Appendix A for the physical reason of such expectation). It is clear then that *in order to estimate accurately the mass of low-mass helium WDs we do need to account for the diffusion processes.*

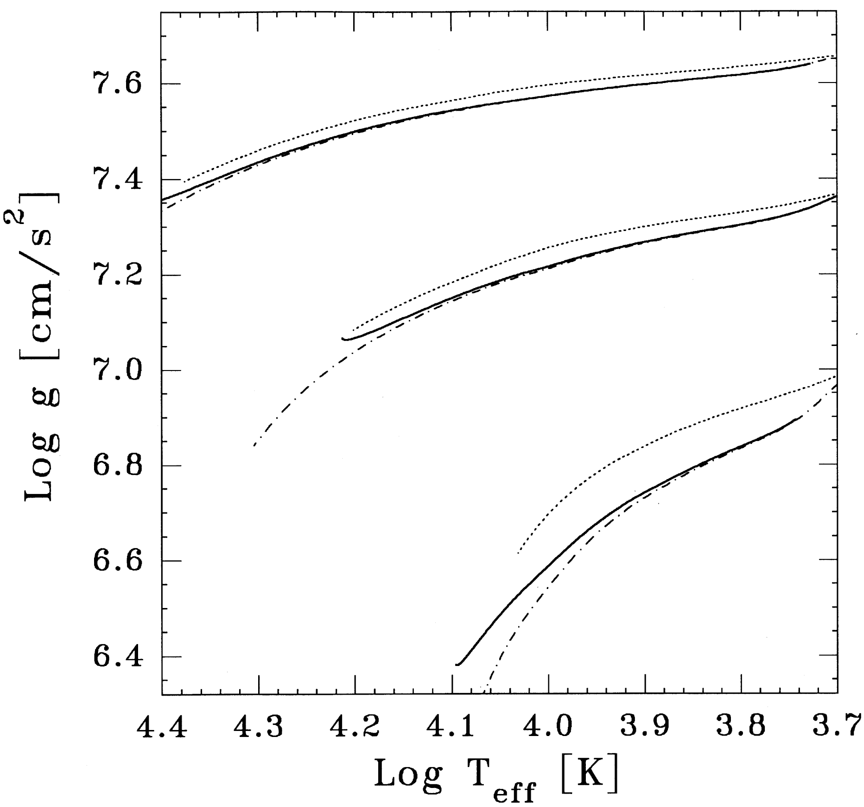

Figure 10. Surface gravity in terms of $\log(T_{\rm eff})$ for, from top to bottom, 0.414-, 0.30- and 0.195-$M_\odot$ models. The results corresponding to the case when diffusion is neglected are shown by dotted lines for models having $M_{\rm env}/M_* = 1\times10^{-3}$ (with a hydrogen content by mass of $X_{\rm H}=0.7$), $M_{\rm env}/M_* = 2\times10^{-3}$ (with $X_{\rm H}=0.7$) and $M_{\rm env}/M_* = 6\times10^{-3}$ (with $X_{\rm H}=0.538$) for the 0.415-, 0.3 and 0.195-$M_\odot$ models, respectively. For the same envelope masses and initial hydrogen content, solid lines correspond to the case when diffusion is included. Finally, dot–dashed lines represent models having pure hydrogen envelopes with the same hydrogen content as in the previous models. Note that the change in surface gravities brought about by diffusion is quite appreciable, particularly for less massive models.

5 SUMMARY

Motivated by recent theoretical predictions about the existence of low-mass, helium white dwarfs (WDs) with envelopes consisting partly of hydrogen and partly of helium, the aim of this work has been to study the effect of diffusion processes on the structure and evolution of such WDs. To this end, we have developed a set of routines to solve the equations describing gravitational settling, thermal and chemical diffusion for a multicomponent plasma. These routines have been coupled to our evolutionary code in order to follow the WD evolution self-consistently. The constitutive physics of our code is based on updated radiative opacities and equations of state for hydrogen and helium plasma. In particular, the energy transport by convection has been treated in the framework of the stellar turbulent convection model developed by Canuto et al. (1996).

In this work we have not intended to perform a binary evolution calculation that leads to the formation of helium WDs (see Sarna et al. 1999ab for recent studies on this subject), and therefore our initial models have been constructed by using an artificial evolutionary procedure, allowing diffusion to operate since the model has relaxed to the cooling track. Needless to say, this is a weak point of our work, so the effective temperature ($T_{\rm eff}$) values at which element separation occurs in our models should be considered as lower limits.

Specifically, we have followed the evolution of helium WD models with stellar masses of 0.414, 0.30 and 0.195 $M_\odot$ under the influence of diffusion down to very low $T_{\rm eff}$ values. For the 0.414-$M_\odot$ models we used an envelope characterized by a mass fraction of $M_{\rm env}/M_* = 1\times10^{-3}$ and a hydrogen abundance by mass of $X_{\rm H} = 0.7$; for the 0.3-$M_\odot$ models we considered $M_{\rm env}/M_* = 2\times10^{-3}$ and $X_{\rm H} = 0.7$ and, finally, for the 0.195-$M_\odot$ models we used $M_{\rm env}/M_* = 6\times10^{-3}$ with $X_{\rm H} = 0.52$. These envelope mass and hydrogen abundance values correspond approximately to those found by DSBH98.

In a set of figures we have explored the evolution of abundance distributions, surface gravities, diffusion velocities and time-scales, and the importance of thermal diffusion in these low-mass WDs has also been addressed. The emphasis was particularly focused on the evolution of the distribution of element abundances and its effects for the structure and evolution of these stars. In this context, we found that, as a result of the hydrogen diffusing upwards, the surface gravity–$T_{\rm eff}$ relation for less massive models is substantially altered from the situation when diffusion is neglected (as proposed in Benvenuto & Althaus 1999). Clearly, diffusion causes the outer layers to become closer to the case of complete separation between hydrogen and helium. It is worth mentioning that only by means of a full simulation including the previous evolutionary stages of the WD shall we be able to make a detailed prediction of the chemical profile of the outer layers. However, the results of the present paper indicate that the assumption we have made in our previous papers on WD structure regarding the purity of the hydrogen and helium layers is largely justified.

In closing, we note that in this paper we have studied the effect of diffusion on the evolution of low-mass helium WDs starting with a hot WD configuration. However, notice that diffusion may have a very important role in determining the previous evolution too. We draw the reader's attention to the remarkable paper of Iben & Tutukov (1986), who considered the process of formation of a ≈0.30-$M_\odot$ helium WD, allowing gravitational settling and chemical diffusion (not thermal diffusion) to operate. The important point is that as diffusion brings fresh hydrogen to very hot layers it may induce the occurrence of a flash, which should decrease the final mass of the outer hydrogen envelope. Thus, because of the present uncertainty in the actual value of the hydrogen mass fraction $M_{\rm H}/M_*$, the estimation of helium WD masses employing the surface gravities available in the literature must be performed with caution.

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APPENDIX A: THE CHANGE OF THE STELLAR RADIUS INDUCED BY A CHANGE IN THE CHEMICAL COMPOSITION – AN APPROXIMATE, ANALYTIC MODEL

In this Appendix we shall present an analytic, approximate model to account for the change in the radius of the WD resulting from a change in the chemical composition of the outer layers. Because this effect seems to have been overlooked and as the computer code is very complicated, we judge that it is very desirable to have a simple model capable of explaining the origin of this effect. In doing so, we shall employ the treatment of the WD outer layers presented in Shapiro & Teukolsky (1983).

Let us assume that the WD envelope is non-degenerate, in radiative equilibrium, and with the opacity given by the Kramer's formula

$$\kappa = 4.34 \times 10^{24} Z(1 + X_{\rm H})\rho T^{-3.5}\,{\rm cm}^2\,{\rm g}^{-1}. \quad \text{(A1)}$$

We assume that this description is valid down to some point at which the non-degenerate electron pressure equals that of the non-relativistic, degenerate electrons. At such a point, the density ρ_* and temperature T_* satisfy

$$\rho_* = 2.4 \times 10^{-8} \mu_{\rm e} T_*^{3/2}\,{\rm g\,cm}^{-3}, \quad \text{(A2)}$$

where $\mu_{\rm e}$ is the mean molecular weight per electron. In this approximation, the WD luminosity may be written as

$$L = 5.7 \times 10^5 \frac{\mu}{\mu_{\rm e}^2} \frac{1}{Z(1 + X_{\rm H})} \frac{M}{{\rm M}_\odot} T_*^{3.5} \quad \text{(A3)}$$

(μ is the mean molecular weight of the plasma). Now, if r_* is the radius corresponding to the bottom of the non-degenerate envelope, and Δr is its thickness,

$$\Delta r = R - r_*, \quad \text{(A4)}$$

then, T_* may be written as

$$T_* = \frac{1}{4.25} \frac{\mu m_{\rm H}}{k_{\rm B}} \frac{GM}{R(R - \Delta r)} \Delta r. \quad \text{(A5)}$$

Let us apply these equations to a somewhat idealized situation. Consider some physical process that changes the chemical composition of the non-degenerate layers (notice that this is the portion of the star in which diffusion is most relevant) in such a way that L remains unchanged.[2] Thus, if the stellar mass and the heavy-element abundances remain constant, equation (A3) tells us that

$$\frac{\mu}{\mu_{\rm e}^2} \frac{1}{1 + X_{\rm H}} T_*^{3.5} \quad \text{(A6)}$$

is invariant. Now, if we consider the case (corresponding to our 0.195-M$_\odot$ model) in which the initial composition is $X_{\rm H} = X_{\rm He} = 0.5$ ($\mu^{\rm i} = 0.7272$, $\mu_{\rm e}^{\rm i} = 1.3333$) and the final composition is $X_{\rm H} = 1$, $X_{\rm He} = 0$ ($\mu^{\rm f} = 0.5$, $\mu_{\rm e}^{\rm f} = 1$), then

$$T_*^{\rm f} = 1.025 T_*^{\rm i}, \quad \text{(A7)}$$

[2] In this way we do not allow the change of radius caused by thermal effects to operate.

i.e. almost unchanged. Thus, this implies that

$$\mu \frac{1}{R(R - \Delta r)} \Delta r \quad \text{(A8)}$$

is almost constant. If we assume that r_* remains constant (which is a good approximation, as suggested by the numerical models), then

$$\mu^{\rm f} \frac{(\Delta r)^{\rm f}}{r_* + (\Delta r)^{\rm f}} = \mu^{\rm i} \frac{(\Delta r)^{\rm i}}{r_* + (\Delta r)^{\rm i}}. \quad \text{(A9)}$$

To a first-order approximation, we find that

$$(\Delta r)^{\rm f} = \frac{\mu^{\rm i}}{\mu^{\rm f}} (\Delta r)^{\rm i}, \quad \text{(A10)}$$

which for this case results in $(\Delta r)^{\rm f} = 1.454(\Delta r)^{\rm i}$, which indicates that the radius of the WD is increased because of diffusion. The resulting change in the surface gravitational acceleration is

$$g^{\rm f} = \left[1 - 2\left(\frac{\mu^{\rm i}}{\mu^{\rm f}} - 1\right) \frac{(\Delta r)^{\rm i}}{R^{\rm i}}\right] g^{\rm i}. \quad \text{(A11)}$$

From the above discussion, it is clear that the key quantity is Δr. Notice, from equation (A5) and for a given value of T_* and μ, that $\Delta r \propto R^2$. However, R is a steeply decreasing function of the stellar mass. This indicates that $\Delta r/R$ will be large only for low-mass WDs. Thus, in order to compute accurate mass–radius relations for WDs, the chemical composition profile of the outer, non-degenerate layer will be relevant only in the case of low-mass objects.

For example, at $\log T_{\rm eff} = 4$, for the 0.195-M$_\odot$ model, $\Delta r/R = 0.34$ and $\Delta \log g = -0.15$. In the case of the 0.414-M$_\odot$ model, at the same $\log T_{\rm eff}$ value, $\Delta r/R = 0.11$, the initial composition is $X_{\rm H} = 0.70$, $X_{\rm He} = 0.3$ ($\mu^{\rm i} = 0.6154$, $\mu_{\rm e}^{\rm i} = 1.1765$) and the final composition is the same as before, then $\Delta \log g = -0.022$. Both cases are in very good agreement with numerical results (see Fig. 10). This justifies the numerical results related to changes in surface gravity resulting from diffusion, presented in the main body of the paper.

Finally, we notice (as is well known) that convection may also produce a change in the chemical composition of the outer layers of a WD. In the case in which hydrogen and helium are separate, if convection reaches deep enough to dredge up helium, it will change the chemical profile by mixing them (i.e. in the opposite direction from that in the case of diffusion). Thus, convective mixing makes the WD radius *decrease*. This effect has been found numerically by Benvenuto & Althaus (1998; see figs 1–12 of that paper). For the reasons given above in this Appendix, it is not surprising that detailed computations indicate this effect to be important only for very low-mass objects. Notice, also, that mixing happens in the case of very low values of the hydrogen mass fraction $M_{\rm H}/M_* \lesssim 10^{-6}$, because only in this case does convection reach helium layers, a necessary condition for mixing to occur.

APPENDIX B: ABOUT OUR INITIAL MODELS

Recently, DSBH98 have criticized our previous works on low-mass WDs. They stated that our procedure for constructing the initial model is not correct.

Obviously, the starter model choice affects the initial evolution of all of our models, particularly the age. For the range of luminosities and $T_{\rm eff}$ values considered in this and our previous papers, however, the starter model choice is no longer relevant and the mechanical and thermal structure of our models are

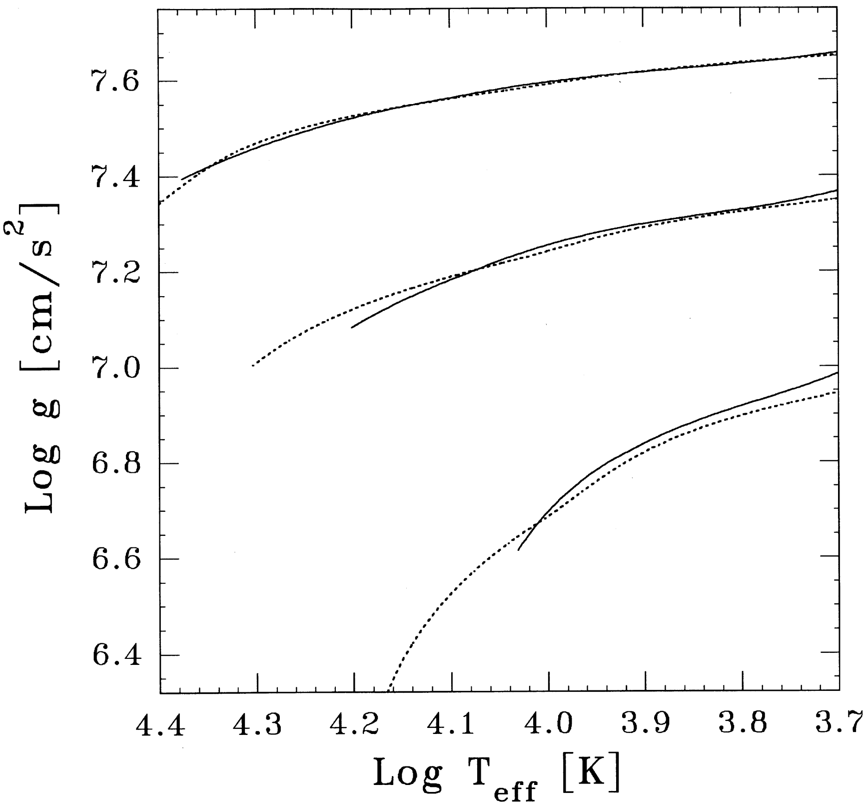

Figure B1. Surface gravity versus $T_{\rm eff}$ for, from top to bottom, 0.414-, 0.30- and 0.195-$M_{\odot}$ helium WD models. Dotted lines correspond to models calculated by DSBH98, while solid lines correspond to our models with $M_{\rm env}/M_{*} = 1\times10^{-3}$ (with a hydrogen content by mass of $X_{\rm H} = 0.7$), $M_{\rm env}/M_{*} = 2\times10^{-3}$ (with $X_{\rm H} = 0.7$) and $M_{\rm env}/M_{*} = 6\times10^{-3}$ (with $X_{\rm H} = 0.538$) for the 0.415-, 0.3- and 0.195-$M_{\odot}$ models, respectively. These envelope mass and hydrogen abundance values correspond approximately to those quoted by DSBH98. Diffusion has been neglected in both sets of models.

completely meaningful. To clarify this point better, we compare the predictions of our models with those of DSBH98 (see also Driebe et al. 1999) who performed a more realistic treatment of the WD progenitor evolution than that we attempt here. To this end, we show in Fig. B1 the surface gravity in terms of $T_{\rm eff}$ for 0.195-, 0.3- and 0.414-$M_{\odot}$ helium WD models. In order to make a direct comparison with DSBH98 predictions, we have adopted for these models the same envelope mass and hydrogen surface abundance as quoted by these authors. Note that, after the relaxation phase of our models, gravities are very similar to those predicted by these authors. It is worth mentioning that DSBH98 have criticized our mass–radius relations in terms of their 'contracting models'. However, such contracting models are very different from our initial ones. In fact, they start with a homogeneous main-sequence model in which nuclear energy release has been suppressed. Therefore, it is not surprising that they obtain contracting models with gravities comparable to those obtained with evolutionary models only when they are very cool (at $T_{\rm eff} \approx 3000$ K for a 0.2-$M_{\odot}$ model). In contrast, both in this and in our previous works on WDs, we generated our initial models from a helium WD model, as explained above. Thus, notwithstanding the comments of DSBH98, our artificial procedure gives rise, under the same physical assumptions, to mass–radius relations in good agreement with those found with a fully evolutionary computation of the stages previous to the WD phase.

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