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Evolution of white dwarfs as a probe of theories of gravitation: the case of Brans–Dicke

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ABSTRACT

Theories with varying gravitational constant G have long been studied. Among them, the most promising candidates as alternatives to standard general relativity are known as scalar–tensor theories. They are consistent descriptions of the observed Universe as well as the low-energy limit of several pictures of unified interactions. Thus, increasing interest in the astrophysical, gravitational wave and pulsar evolution consequences of such theories has been sparked over the last few years. In this work we study the evolution of white dwarf stars in the framework of the simplest model of scalar–tensor theory: Brans–Dicke gravity. We assume that the star is able to *see* the cosmological evolution of G (obtained from relativistic equations) while adopting a Newtonian model for describing its structure. This allows us to determine how the G variation affects the energetics of the stellar interior. The white dwarfs are analysed employing a well-tested computer code, with state-of-the-art data for the equation of state, opacities, neutrinos, etc.; all these characteristics are carefully described in the text. We compute the theoretical white dwarf luminosity function and use previous observational data to compare with and extract conclusions on the feasibility of the gravitational theory analysed. We find several striking results. The cooling of white dwarfs is strongly accelerated, particularly for massive stars and low luminosities, even if the ω parameter of Brans–Dicke theory is big enough to accord well with any other test of gravitation. This uncommon cooling process translates into several distinctive features of white dwarf evolution, among which are (a) a new profile of luminosity versus fractional mass and age, (b) different central temperature versus surface luminosity, (c) low masses of progenitors, and most importantly (d) an appreciable variation in the luminosity function. We finally analyse the possibilities of, when precise data with unique interpretation are available, converting this into a powerful new test of gravitation.

Key words: gravitation – methods: analytical – stars: luminosity function, mass function – white dwarfs – cosmology: theory.

1 INTRODUCTION

The idea of a varying gravitational constant G has been in physicists' minds for a long time since Dirac (1937a,b) [see also Chandrasekhar 1937; Kothari 1938; Teller 1948; and see Barrow & Tipler (1986) for a comprehensive analysis of variation in the fundamental constants] proposed what was called the *large number hypothesis*. It states that the ubiquity of some large dimensionless numbers of order 10^{40}, arising from the combination of micro- and macrophysical parameters, was not a coincidence but a result of an underlying time variation in the combination $e^2G^{-1}m_{\rm p}$, where e is the unit charge and $m_{\rm p}$ is the proton mass. Dirac himself ascribed to a varying G and wrote this time variation into Newtonian expressions. However, well-posed gravitational theories admitting a variation in the fundamental constants had to wait for about twenty years after these ideas, being mainly introduced by Brans & Dicke (1961). They developed a relativistic theory of gravity based on the existence of a scalar field in a Riemannian geometry. The gravitational *constant* was reborn as a field variable by the assignment $G^{-1} = \phi(\boldsymbol{x})$ (below we shall see this in further detail). Excluding some philosophical arguments concerning the applicability of Mach's and Berkeley's visions of the Universe, one of the first motivations for the replacement of general relativity

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(GR) by a Brans–Dicke (BD) or generally by a scalar–tensor (ST) theory was a seeming discrepancy between observations and the weak-field GR predictions in the Solar system. Time went by and these differences vanished, making the main motivation for these gravitational arenas shift into a cosmological scenario.

In ST theories, the gravitational action has a free parameter $\omega(\phi)$, called *coupling function* because it measures the strength of the coupling between the field and the metric. For particular values of this parameter, these theories have cosmological solutions that are entirely compatible with every gravitational test. Solar system tests (Reasenberg & Shapiro 1976; Reasenberg et al. 1979; Will 1993), gravitational lensing (Krauss & White 1992; Bekenstein & Sanders 1994) and nucleosynthesis (Domínguez-Tenreiro & Yepes 1987; Accetta, Krauss & Romanelli 1990; Casas, García-Bellido & Quirós 1992; Torres 1995; Kalligas, Nordvedt & Wagoner 1997; Damour & Pichon 1998) are some of the frameworks where their predictions were satisfactorily confronted with obtained data. Still, as we shall soon see, these theories may have a crucial role in the early Universe. The scalar field ϕ is a possible source for inflation (Mathiazhagen & Johri 1984; La & Steinhardt 1989; Steinhardt & Accetta 1990; Holman et al. 1991; Barrow & Maeda 1992; García-Bellido, Linde & Linde 1994; Barrow 1995) and would modify any form of expansion arising from the existence of other inflations. Moreover, they are the low-energy limit of string theories (Fradkin & Tseytlin 1985; Callan et al. 1985; Lovelock 1985) and scalar fields appear in multidimensional reduction (Berkin & Hellings 1994; Rainer & Zhuk 1996).

In Table 1 we show some examples of the bounds on $\dot{G}/G$ [for a recent confrontation between varying constants, theories and experiments see the article by Will (1998)]. Many of them have, however, several caveats. The knowledge of the asteroid belt is not enough to determine precisely its influence on some of the bounds. The proper motion of a pulsar may affect other quoted numbers. Cosmological nucleosynthesis is strongly model-dependent, as can be seen from the appearance of the Hubble constant H. White dwarf (WD) star cooling (a bound that will be discussed below) and Bayesian statistical techniques applied to neutron masses may be affected by a quasi-Newtonian treatment of $G(t)$. If the variation of G is coupled with variation of other parameters as well, the limits are more uncertain (Sisterna & Vucetich 1990a,b; Vucetich 1996). In general, the conclusion is that, albeit severely constrained, a very slow variation of G cannot be discarded, especially when cosmological time intervals are considered (Barrow 1987); a variation of the fundamental constants of nature cannot be ruled out. An example of this is the recent striking result concerning possible time or space variation of the fine-structure constant (Webb et al. 1999).

ST theories give a consistent frame in which to study a G variation. In the weak-field limit, Nordvedt (1968) found an expression for the observed value of the gravitational *constant*,

$$G(t) = \phi^{-1}\left(\frac{4+2\omega(\phi)}{3+2\omega(\phi)}\right). \quad (1)$$

This yields

$$\frac{\dot{G}(t)}{G} = -\dot{\phi}\left(\frac{3+2\omega(\phi)}{4+2\omega(\phi)}\right)\left(G+\frac{2\dot{\omega}}{(3+2\omega)^2}\right). \quad (2)$$

Thus, the observational limit implies that if $\omega\rightarrow\infty$ and $\dot{\omega}\omega^{-3}\rightarrow 0$ when $t\rightarrow\infty$, the weak-field predictions for the present time will accord well with the standard values. However, this does not preclude that the theory may have significant deviations from GR at early cosmological times, nor that a very slow variation may have occurred during the matter era.

Having arrived at this point, the scientific interest in ST theories split into two (although not unique) basic branches. On one side, it is now very important to investigate relativistic cosmological models and to classify the different varieties of expanding universes and of inflationary scenarios. This is necessary to explore their possible

Table 1. Some bounds on $\dot{G}/G$ [yr^{-1}].

Method/Object	$\dot{G}/G < \ldots$	Reference
Cosmological Nucleosynthesis	$0.01\,H$	1
Radar Data (Mercury-Venus)	4×10^{-10}	2
Radar Data (Mars: Mariner 9)	1.5×10^{-10}	3
Radar Data (Mars: Mariner 10)	$0.0\pm2.0\times10^{-12}$	4
Radar Data (Mars: Viking)	3×10^{-11}	5
Radar Data (Mars: Viking)	$2\pm4\times10^{-12}$	6
Radar Data (Mars: Viking)	$-2\pm10\times10^{-12}$	7
Binary Pulsar (PSR 1913+16)	$-1.10\pm1.07\times10^{-11}$	8
Pulsar-White Dwarf System (PSR B1855+09)	$-9\pm18\times10^{-12}$	9
White Dwarfs Cooling (See discussion below)	$-1\pm1\times10^{-11}$	10
Lunar Occultations and Eclipses	0.4×10^{-10}	11
Laser Ranging Data (Moon)	0.3×10^{-10}	12
Solar Evolution	$\sim10^{-10}$	13
Neutron Star Masses and Ages	$-0.6\pm4.2\times10^{-12}$	14
Paleontological Evidence	$\sim10^{-11}$	15
Stability of Clusters	$5\pm1\times10^{-11}$	16

References – (1) Domínguez-Tenreiro & Yepes (1987); Accetta, Krauss & Romanelli (1990); Casas, García-Bellido & Quirós (1992); Torres (1995); Kalligas, Nortvedt & Wagoner (1997); (2) Shapiro et al. (1971); (3) Anderson et al. (1978); (4) Anderson et al. (1991); (5) Hellings et al. (1989); Reasenberg (1983); (6) Hellings et al. (1983); (7) Dickey, Newhall & Williams (1989); Shapiro (1990); (8) Damour, Gibbons & Taylor (1988); (9) Kaspi, Taylor & Ryba (1994); (10) García-Berro et al. (1995); (11) Morrison (1973); (12) Williams, Sinclair & Yoder (1978); (13) Chin & Stothers (1976); (14) Thorsett (1996); (15) Sisterna & Vucetich (1990b); (16) Deaborn & Schramm (1974).

consequences upon the cosmic microwave background, nucleosynthesis, gravitational radiation and the spectrum of primordial perturbations that give rise to the cosmic structure. To do this, it is desirable to have exact cosmological solutions of the field equations (we shall present these equations below). Recently, a great improvement in the search for these solutions has been given in the form of suitable changes of variables. Barrow (1993) presented a method that enables exact solutions to be found for vacuum- and radiation-dominated Friedmann universes of all curvatures in arbitrary ST theories. Then, and also for arbitrary $\omega(\phi)$, Barrow & Mimoso (1994) and Mimoso & Wands (1995) derived exact Friedmann–Robertson–Walker (FRW) cosmological solutions in models with a perfect fluid satisfying the equation of state $p = (\gamma - 1)\rho$ (with γ a constant and $0 \leq \gamma \leq 2$). Application of these methods to a wide range of couplings was recently made by Barrow & Parsons (1997). For more complicated Lagrangians (with two free functions) these methods were also generalized (Torres & Vucetich 1996; Torres 1997a,b). By no means, however, are these the only solutions to these theories, a subject that has been investigated for more than forty years.

The other branch of research is to try to determine what happens to astrophysical objects if G is a varying function. Most of the work in this area has been focused on black holes and their thermodynamics (see e.g. Campanelli & Lousto 1993; Kang 1996; Kim 1997). Previously, Hawking (1972) had proven that stationary black hole solutions remain unchanged in ST gravity. On usual star models, and as far as we are aware, Salmona (1967) was the first to present a theory of stellar structure in the presence of a scalar field, showing that these theories are compatible with the usual concepts. A number of other authors have investigated fluid spheres, incompressible fluids and high-density situations (Hillebrandt & Heintzmann 1974; Bruckman & Kazes 1977). The gravitational collapse of a gaseous sphere (Thorne & Dykla 1971; Matsuda & Nariai 1973) and the frequency shift of the radiation of a collapsing object have been studied as well. Recently, Harada (1998) has studied neutron stars, examining their stability. Their collapse to a black hole and the process of scalarization was analysed by Novak (1998a,b). The possibility of hiding in the gravitational scalar the violation of the null energy condition necessary to support a wormhole throat has also sparked interest (Anchordoqui, Perez Bergliaffa & Torres 1997; Anchordoqui, Grunfeld & Torres 1998; Nandi et al. 1998).

Probably the most dramatic effect of a varying G is the concept of gravitational memory (Barrow 1992, 1994). Barrow presented these ideas by posing the following problem: What happens to black holes during the subsequent evolution of the Universe if the gravitational coupling G evolves with time? He envisaged two possible scenarios. In *scenario A*, the black hole evolves quasi-statically, in order to adjust its size with the changing G. If true, this means that there are no static black holes, even classically, during any period in which G changes. In the alternative possibility, *scenario B*, the local value of G within the black hole is preserved, while the asymptotic value evolves with a cosmological rate. The black hole *remembers* the strength of gravity at the moment of its formation. Further analysis of the striking phenomena that arise in both of these scenarios was made by Barrow & Carr (1996). Being so general, the same scenarios may concur for any long-lived object like a cosmic string or even the usual hydrogen-burning stars. These concepts were also thoroughly analysed within boson star models, where it was explicitly shown that the variation of G modifies the equilibrium structure of the stars (Torres 1997a; Torres, Liddle & Schunck 1998a; Torres, Schunck & Liddle 1998b).

When one considers a WD star, one has to deal with the fact that their equilibrium configurations do not arise from the relativistic field equations, contrary to the boson star case. WDs are not fully relativistic objects and the variation of G must be imposed in a quasi-Newtonian way. Attempts to ascertain the evolution and cooling processes of WDs in varying-G theories was a subject started by Vila (1976) and recently continued by García-Berro et al. (1995). Whilst understanding that a comprehensive study of the effects of a varying-G cosmology upon astrophysical objects is far from being complete, it is the aim of this work to study in detail the conclusions that may be obtained from WD physics. The time variation of the Newton constant was also proposed as a possible explanation of the discrepancy between the cosmic expansion age and the globular cluster age (Degl'Innocenti et al. 1996).

To be specific, we shall study the problem of the evolution of WD stars in the frame of a specific theory of gravitation with a varying G: Brans–Dicke. We shall compute the evolution of WDs employing a fully detailed and updated computer code, modified in order to include the effects induced by a varying G. After assuming a value for the free parameter of the theory, this will allow us to construct sequences of evolution for WD models of different masses and birth epoch. Having these sequences we are in a position to compute the *theoretical* WD luminosity function (the number of WDs with a given luminosity per cubic parsec and per unit of luminosity; hereafter WDLF). At present we know the *observational* WDLF, and this will allow us to perform a detailed comparison between them. In this way we shall show that WDs are extremely sensitive to a varying G value, and thus very powerful at imposing bounds on the free parameters that fix the variation of G and even discriminating between $\dot{G} = 0$ and $\dot{G} \neq 0$ theories.

Although we refer the reader to the next sections of this work for details, we may give a simple physical reason for the sensitivity of WD evolution to varying-G theories of gravitation. WDs have a rather compact structure, and thus a relatively large gravitational binding energy. Comparatively, they have a low thermal content (relic of the thermonuclear reacting interior of the WD progenitor), which implies a low luminosity. As the gravitational binding energy is proportional to the G value, it is clear that a minute variation of G may even be able to dominate the WD energetic balance over its internal thermal content. As a result, the cooling process will be dramatically accelerated.

The rest of this paper is organized as follows: In Section 2, we present the main characteristics of ST theories as a class of gravitational theories. In Section 3, we describe the modifications to the equation of energy conservation in the case of running G, the numerical code we employed and also the initial models of WDs from which we start our evolutionary sequences. In Section 4, we comment on the procedure we follow to derive the theoretical WDLF. The numerical results found in this study are presented in Section 5. Finally, Section 6 is devoted to discussing the implications of the present analysis on Brans–Dicke theory of gravitation.

2 SCALAR–TENSOR THEORIES: BASIC FORMALISM

There are, in fact, unlimited possibilities for constructing theories of gravity, which involve a metric, matter fields and a scalar field; but a reduced set is obtained if we require that they be deducible from a least-action principle and described by second-order differential equations (Bergmann 1968). From this group, ST gravity has the action

$$S = \int \frac{\sqrt{-g}}{16\pi} \, dx^4 \left[\phi R - \frac{\omega(\phi)}{\phi} \partial_\mu \phi \, \partial^\mu \phi + 16\pi \mathcal{L}_m \right]. \tag{3}$$

Here, as usual, $g_{\mu\nu}$ is the metric, R is the scalar curvature, ϕ is the Brans–Dicke (BD) field, $\mathcal{L}_m$ is the Lagrangian of the matter content of the system and $\omega(\phi)$ is the coupling. The special case of a constant ω is BD theory, and general relativity is regained (although this still needs further analysis for some particular situations) in the limit of large ω. The gravitational constant G of the Einstein–Hilbert action is replaced by a dynamical field, ϕ^{-1}. This enables one to make a proper study of a varying gravitational strength.

Varying the action with respect to the dynamical variables $g^{\mu\nu}$ and ϕ we obtain the field equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi}{\phi} T_{\mu\nu} + \frac{\omega(\phi)}{\phi} \left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2} g_{\mu\nu} \phi^{,\alpha}\phi_{,\alpha} \right) + \frac{1}{\phi^2} \left(\phi_{,\mu;\nu} - g_{\mu\nu} \Box\phi \right), \tag{4}$$

$$\Box\phi = \frac{1}{2\omega + 3} \left[8\pi T - \frac{d\omega}{d\phi} \phi^{,\alpha}\phi_{,\alpha} \right], \tag{5}$$

where we have introduced $T_{\mu\nu}$ as the energy–momentum tensor for matter fields and T as its trace.

We first assess the likely cosmological variation of ϕ, concentrating on BD theory. That is, we are interested in FRW solutions all along the cosmic eras. During radiation domination, $T = 0$, and the attractor behaviour is actually exactly that of general relativity, namely a constant ϕ and $a \propto t^{1/2}$. Here, a is the scale factor in a flat FRW metric. This changes with the onset of matter domination, when the attractor solution becomes (Nariai 1969; Gurevich, Finkelstein & Ruban 1973)

$$a(t) \propto t^{(2-n)/3}; \qquad G(t) \propto t^{-n}, \tag{6}$$

where $n = 2/(4 + 3\omega)$, thus G exhibits a slow decrease. Assuming that matter–radiation equality took place near the general relativity value[1] at $z_{eq} = 24\,000\,\Omega_0 h^2$, then, for critical density and $h = 0.5$, the fractional change in G since equality is

$$\frac{G(t_0)}{G(t_{eq})} = 6000^{-1/(1+\omega)}. \tag{7}$$

For $\omega = 400$, the ratio is 0.98, so G will have changed value by about 2 per cent since equality. This may differ of course for other values of ω. We recall, however, that weak-field tests imply $|\omega| > 500$ (Reasenberg & Shapiro 1978; Reasenberg et al. 1979; Will 1993). Thus, in the last gigayears of the Universe the value of G has stayed almost constant. Whether this very slow variation is enough to produce a visible stellar signal will be explored in the rest of this work. Before going to the results, we shall present how the simulations were done and which was the gravitational input.

3 THE EVOLUTION OF WHITE DWARF STARS WITH RUNNING G

We shall assume that BD theory (with different values of ω) is the correct theory of gravity and shall follow the evolution and cooling of WDs through the matter era. The value of G will be fixed at every time by the cosmological evolution of the BD theory, the solutions of which were given above. Thus, we have a quasi-Newtonian treatment of G. Its actual value is obtained from relativistic cosmological equations but the stellar equations will be in the Newtonian limit. If we recall the scenarios presented in the introduction, we shall analyse both of them: the case in which the star remembers a fixed value of G, that valid at its formation; and the case in which the star is able to *see* the cosmological evolution of G and its value changes with time.

The problem of the evolution of WD stars in the case of varying G was previously addressed by Vila (1976) and by García–Berro et al. (1995). Vila (1976) employed a rough analytical treatment of the problem that, coupled to the lack of knowledge of the faint portion of the WDLF at that time, prevented him from reaching a firm constraint to $\dot{G}$. Much more recently, García–Berro et al. (1995) considered the WD evolution assuming an exponential variation of G along all the evolution, i.e. $\dot{G}/G =$ constant. They were the first to establish upper bounds to the rate of change of G by employing the observed WDLF. However, they employed a very simplified (semi-analytic) treatment for the WD by describing its evolution in terms of an isothermal interior, and by neglecting the effect of the variation of G in the structure of the stellar envelopes they considered. Finally, they only considered the case of $M_* = 0.60\,M_\odot$ at the moment of comparing their results with the observed WDLF. In spite of the approximations involved in such an approach, it is remarkable that García–Berro et al. (1995) have found a very restrictive bound, comparable to the best presently available (see Table 1).

3.1 The equation of energy conservation in the case of a running G

In this subsection, we shall deduce the equation of energy conservation in the presence of a running G value. We shall proceed following closely the treatment given in Kippenhahn & Weigert (1990). Our approach at this stage follows that of Garcia-Berro et al. (1995) [i.e. see our equation (19) and their equation (1)].

For the whole star we may write the conservation of energy as

$$\frac{d}{dt}(E_n + E_i + E_g) + L + L_\nu = 0, \tag{8}$$

where E_n, E_i and E_g are the nuclear, internal and gravitational energies, and L and L_ν are the photon and neutrino luminosities respectively. The only term in which G appears is E_g, and thus it will give a contribution proportional to $\dot{G}/G$. The other terms may be handled in the standard way, and we refer the reader to the book of Kippenhahn & Weigert (1990, p. 23) for details.

It is well known that E_g may be written as

$$E_g = -\int \frac{GM_r}{r} dM_r = -3 \int \frac{P}{\rho} dM_r, \tag{9}$$

where M_r is the mass enclosed in a sphere of radius r, P is the pressure and ρ is the mass density. The integral runs over the whole stellar interior. Thus, $\dot{E}_g$ may be written as

$$\dot{E}_g = -3 \int \left(\frac{\dot{P}}{\rho} - \frac{P}{\rho^2}\dot{\rho} \right) dM_r. \tag{10}$$

In Lagrangian coordinates, the equation of hydrostatic equilibrium reads

$$\frac{\partial P}{\partial M_r} = -\frac{GM_r}{4\pi r^4} \tag{11}$$

thus $\partial\dot{P}/\partial M_r$ is

$$\frac{\partial \dot{P}}{\partial M_r} = \left(\frac{\dot{G}}{G} \right) \frac{\partial P}{\partial M_r} + \frac{GM_r}{\pi r^5} \dot{r}. \tag{12}$$

[1] Although this assumption is very well justified (see Liddle, Mazumdar & Barrow 1998), the exact value cannot be computed analytically (Torres & Helmi 1996).

If we multiply by $4\pi r^3 dM_r$ and integrate, the first term on the RHS of equation (12) is

$$\left(\frac{\dot{G}}{G}\right)\int 4\pi r^3 \frac{\partial P}{\partial M_r}\, dM_r = \left(\frac{\dot{G}}{G}\right)E_g, \qquad (13)$$

whereas the LHS of equation (12) can be reduced to

$$\int 4\pi r^3 \frac{\partial \dot{P}}{\partial M_r}\, dM_r = -3\int \frac{\dot{P}}{\rho}\, dM_r \qquad (14)$$

and thus

$$-3\int \frac{\dot{P}}{\rho}\, dM_r = \left(\frac{\dot{G}}{G}\right)E_g + 4\int \frac{GM_r}{r^2}\, \dot{r}\, dM_r. \qquad (15)$$

But $\dot{E}_g$ may also be expressed as

$$\dot{E}_g = \left(\frac{\dot{G}}{G}\right)E_g + \int \frac{GM_r}{r^2}\, \dot{r}\, dM_r. \qquad (16)$$

From the last two equations we obtain

$$-3\int \frac{\dot{P}}{\rho}\, dM_r = 4\dot{E}_g - 3\left(\frac{\dot{G}}{G}\right)E_g. \qquad (17)$$

If we replace this in equation (10), we find that $\dot{E}_g$ can also be written as

$$\dot{E}_g = -\int \frac{P}{\rho^2}\, \dot{\rho}\, dM_r + \left(\frac{\dot{G}}{G}\right)E_g. \qquad (18)$$

The first term on the RHS of equation (18) is the standard one, whereas the second term arises from the assumption of a running G value. Consequently, in a differential form, the equation of conservation of energy for a running G value (apart from a contribution from the release of latent heat during crystallization) is

$$\frac{\partial L_r}{\partial M_r} = \varepsilon - \varepsilon_\nu - C_p\frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} + \left(\frac{\dot{G}}{G}\right)\frac{GM_r}{r}, \qquad (19)$$

where ε is the nuclear energy release, ε_ν is the energy lost in neutrino emission, C_p is the specific heat at constant pressure and $\delta = \partial \ln\rho/\partial \ln T|_P$.

The last term in equation (19) describes the change in the gravitational binding energy of the star as a result of the change in the G value. If we assume some theory of gravitation that predicts $\dot{G} \neq 0$, the gravitational binding energy of the star is so large that, even in the case of very slow variations, their product makes a non-negligible contribution to the energy balance of the star. It is worth mentioning that the remaining equations of stellar structure and evolution have their usual form, but if we assume (as done in most of this work) that the star is able to feel the varying G value, we need to consider the particular G value corresponding to the cosmological age. It is important to note that, if the WD is able to remember its G value at birth, its evolution would be hardly distinguishable from that corresponding to the standard case with $\dot{G} = 0$.

The net effect of the term including $\dot{G}$ is simple: as a consequence of the decrease in G, the star inflates and it absorbs a large amount of energy that otherwise would have been released as electromagnetic radiation. Consequently, the process of cooling is strongly accelerated. This effect is more noticeable the lower the value of ω (i.e. the larger is the value of $-\dot{G}$) and the earlier the WD birth occurs.

An interesting consequence of such a term is that the evolution of a star depends on the time of its birth, which is not the usual case. This is simply because the function describing the trend of $\dot{G}$ is dependent upon the cosmological time and not upon the stellar evolution one. In other words, the *differential equations of stellar evolution are no longer invariant under temporal translations*. This is not a minor effect, as will be made clear below.

3.2 The numerical code and initial models

The WD evolutionary code we employed in this study has been used to study different problems connected with WD evolution, and it is fully described in Althaus & Benvenuto (1997), Benvenuto & Althaus (1997, 1998) and references cited therein. The code is based on the technique developed by Kippenhahn, Weigert & Hofmeister (1967) for calculating stellar evolution. In particular, to specify the surface boundary conditions we carry out three envelope integrations (at constant luminosity) from photospheric starting values inward to a fitting mass fraction $M_1/M_* \approx 10^{-16}$, where M_1 corresponds to the first mass shell and M_* is the total mass of the model. In our code the value of M_1 is automatically changed over the evolution so as to keep the thickness of the envelope as small as possible. This provides an accurate description of the outer layers of our WD models. The interior integration is treated according to the Henyey iterative scheme as described by Kippenhahn et al. (1967).

To compute the evolution of WDs in the frame of the Brans–Dicke theory of gravitation, we carried out some modifications to our code in order to take into account the variation of the value of G self-consistently. In particular, the full system of differential equations of stellar evolution are now solved considering the equation for the energy balance given by equation (19). To our knowledge, this is the first time the evolution of WD stars with a running value for the gravitational coupling constant G has been solved in this way.

It is worth mentioning that our code is based on a detailed and updated constitutive physics appropriate to WD stars. Briefly, the equation of state for the low-density regime is that of Saumon, Chabrier & Van Horn (1995) for hydrogen and helium plasmas. The treatment for the completely ionized, high-density regime includes ionic contributions, Coulomb interactions, partially degenerate electrons and electron exchange and Thomas–Fermi contributions at finite temperature. Radiative opacities for the high-temperature regime ($T \geq 6000$ K) with metallicity $Z = 0$ are those of OPAL (Iglesias & Rogers 1993), whilst for lower temperatures we use the Alexander & Ferguson (1994) molecular opacities. Conductive opacities and the various mechanisms of neutrino emission relevant to WD interiors are taken from the works of Itoh and collaborators (see Althaus & Benvenuto 1997 for details). With regard to the energy transport by convection, we adopt the mixing length prescription usually employed in most WD studies. Finally, we consider the release of latent heat during crystallization in the same way as in Benvenuto & Althaus (1997).

As mentioned previously, we computed the evolution of models with masses ranging from $M = 0.4$ to $1.0\,M_\odot$ with a mass step of $0.1\,M_\odot$ and with a hydrogen envelope mass of $M_H/M_* = 10^{-5}$. The grid in stellar mass will allow us to construct accurate WDLFs for each pair of values (ω, t_U) where t_U is the age of the Universe. The initial models were obtained following the artificial evolutionary procedure described in our previous papers cited above. We mention that we consider the same interior composition of carbon and oxygen for models of different masses, notwithstanding the changes that are expected to occur as a result of the differences in the pre-WD evolution. The carbon–oxygen core of our models is surrounded by an almost pure helium envelope, the mass of which is taken as $M_{He}/M_* = 0.01$. In DA WDs [a class of WDs that show (almost only) hydrogen lines in its spectrum], there is an almost pure hydrogen envelope on the top of the helium layers. Unfortunately, the mass of this hydrogen envelope is only weakly constrained by pre-WD evolutionary calculations (D'Antona &

Mazzitelli 1991). In recent years, however, strong evidence has been accumulated favouring the idea that some ZZ Ceti stars (variable DA WDs) appear to have thick hydrogen layers (Fontaine et al. 1994). In the present study, we adopt a rather thick hydrogen envelope ($M_H/M_* = 10^{-5}$) that is nevertheless thin enough to neglect any nuclear burning at its bottom. Simultaneously, it is thick enough to ensure that the outer convection zone never reaches its bottom, preventing any mixing with the underlying helium layer from occurring. Thus, our WD models undergo no change of the chemical profile throughout their entire evolution. Finally, we assumed the hydrogen/helium transition zone to be almost discontinuous.

In order to take into account the dependence of the WD evolution on the age of the Universe t_U and, more importantly, on the time of its birth t_B (see Section 3), we have calculated evolutionary sequences for values of $t_U = 7.5$, 10 and 12.5 Gyr, and for $t_B = t_{MS}, t_U/2 + t_{MS}$ and $t_U + t_{MS}$, where t_{MS} is the approximate time spent by the WD progenitor in forming a WD star. For each set of values (t_U, t_B, M_*) we computed the evolution considering for the constant ω the values 400, 600, 800, 1000, 2000, 5000 and 10 000.

4 THE WHITE DWARF LUMINOSITY FUNCTION

Over the last two decades, the possibility of using the observed WDLF in understanding crucial aspects of Galactic disc history has captured the interest of numerous investigators. Indeed, D'Antona & Mazzitelli (1978) (see also Schmidt 1959 for an earlier reference) pointed out that a deficiency of observed WDs at low luminosities (established by Liebert et al. 1979) could simply be ascribed to a finite age of the Galactic disc. On the other hand, detailed evolutionary calculations of WD evolution (Iben & Tutukov 1984; D'Antona & Mazzitelli 1989) showed that the Debye cooling is not sufficiently rapid to give rise to such a paucity of faint WDs. The idea that the drop in the WDLF be interpreted in terms of the finite age of the Galactic disc has been extensively and quantitatively explored by many investigators such as Winget et al. (1987), García-Berro et al. (1988), Iben & Laughlin (1989), Yuan (1989), Noh & Scalo (1990), Wood (1992), Oswalt et al. (1996) and Leggett, Ruiz & Bergeron et al. (1998). By fitting the observations with theoretical WDLFs, these authors have studied the observed WDLF with the major aim of obtaining information about the age and star formation history of the Galaxy.

The existence of an abrupt fall-off in the WDLF was placed on a firm observational basis by Liebert, Dahn & Monet (1988), who established that the downturn in the WDLF takes place at $\log(L/L_\odot) \approx -4.4$ and it is not a result of some selection effect. Recently, Oswalt et al. (1996) presented an independent determination of the WDLF based on a sample of WDs in binary systems. In particular, the procedure adopted by Oswalt et al. for correcting against sample incompleteness gives rise to an effective search volume far greater than that considered in previous WD surveys (see also Wood & Oswalt 1998). In addition, the number of faint WDs suggested by the lowest luminosity bin used in their study is five to six times larger than that encountered by Liebert et al. (1988). These features together with updated WD evolutionary calculations for DA WD models (Wood 1995) suggest an age for the local Galactic disc of ≈9.5 Gyr.

Much more recently, Leggett et al. (1998) have substantially improved the determination of the observed WDLF for the cool objects in the sample of Liebert et al. (1988). They employed state-of-the art model atmospheres to compute accurate bolometric corrections, and find, on the basis of the WD models of Wood (1995), an age for the Galactic disc of ≈8.0 Gyr. Notably, the last bin in the Leggett et al. (1998) WDLF results in a spatial density of dim WDs about 20 times lower than in the work by Oswalt et al. (1996). As argued by Leggett et al. (1998), their WDLF at low luminosities is better determined than that of Oswalt et al. For this reason, we shall compare our results with the Leggett et al. WDLF.

To compare with the observed space density of WDs, we construct integrated WDLFs from our evolutionary sequences. To this end, we closely follow the treatment presented in Iben & Laughlin (1989) (see also D'Antona & Mazzitelli 1978). Specifically, the space density of WDs per unit of $\log(L/L_\odot)$ is calculated from

$$\frac{\mathrm{d}n}{\mathrm{d}\log L/L_\odot} = -\psi_o \int_{M_i}^{M_s} \Phi(M)\ \frac{\mathrm{d}t_{cool}}{\mathrm{d}\log L/L_\odot}\ \mathrm{d}M. \qquad (20)$$

Here, $\Phi(M)$ is the Salpeter initial mass function of WD progenitors with stellar mass M (which predicts that the created star distribution is proportional to $M^{-2.35}$) and t_{cool} is the WD cooling time at a given $\log(L/L_\odot)$. M_i and M_s denote respectively the minimum and the maximum mass of the main-sequence stars that contribute to the WD space density at $\log(L/L_\odot)$. We take $M_s \approx 8\,M_\odot$ (Wood 1992) and M_i is obtained by considering the earliest period of star formation in the Galactic disc, that is by solving the equation

$$t_{MS}(M) + t_{cool}(\log(L/L_\odot), M_*) = T_{disc},$$

where T_{disc} is the assumed disc age (see Iben & Laughlin 1989 for details). Here, the pre-WD evolutionary times $t_{MS}(M)$ are those[2] of Iben & Laughlin (1989). Since for the case of our stellar models with varying G, t_{cool} depends not only on the WD mass but also on the time of birth t_B of the WD, we use, in order to determine M_i, linear interpolation between the t_{cool} values corresponding to those sets of evolutionary sequences whose t_B values (for a given t_U) bracket the value $t_B = t_{MS} + t_U - T_{disc}$, i.e. $t_B = t_{MS}$, $t_{MS} + t_U/2$ (we assume $T_{disc} = 0.8 t_U$). As far as the initial (M)–final (M_*) mass relation is concerned, we use an exponential model: $M_* = 0.40\exp(0.125M)$ (Wood 1992). In deriving equation (20), the star formation rate ψ_o has been assumed to be constant, an approximation that does not affect the main conclusions of this work. Finally, for each of the selected luminosity values, we calculate $\mathrm{d}t_{cool}/\mathrm{d}\log L/L_\odot$ at the required values of M_* and t_B (such that $t_B + t_{cool} = T_U$) by using linear interpolation between the $\mathrm{d}t_{cool}/\mathrm{d}\log L/L_\odot$ values of the sequences that bracket M_* and t_B. This is done because the WDLF is observed at the present cosmic time, when $G = G_0$. It is worth mentioning that all of our theoretical curves have been normalized to the observed space density of 0.003 39 WDs per cubic parsec (Leggett et al. 1998) (this means that the integration of the WDLF over the entire range of luminosities yields the observed space density, thus fixing ψ_o).

5 NUMERICAL RESULTS

We present now the main results of our calculations. As stated earlier, we computed the evolution of carbon–oxygen DA WD models with masses ranging from $M = 0.4\,M_\odot$ to $M = 1.0\,M_\odot$ at intervals of 0.1 $M_\odot$ and with a metallicity of $Z = 0$. We assumed the

[2] We warn the reader that these times have been computed assuming a constant G value. In spite of this, we judge that this should introduce a minor effect on the conclusions of the present work, simply because in the pre-WD stages, the star has a larger size, and thus the effects induced by the presence of the last term in equation (19) should be of minor importance.

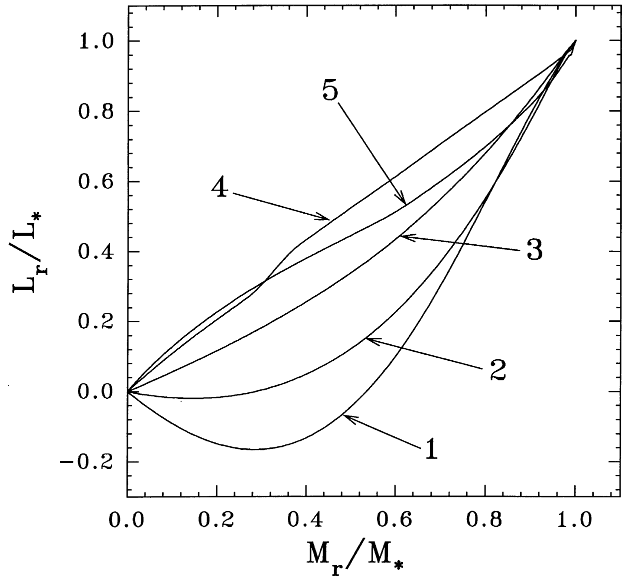

Figure 1. Profile of relative luminosity versus fractional mass for a 0.4-M$_\odot$ WD model calculated assuming $\omega = 600$, $t_{\rm U} = 12.5\,$Gyr and $t_{\rm B} = t_{\rm MS}$ (see text for explanation). Curves labelled as 1, 2, 3, 4 and 5 correspond respectively to $\log L/{\rm L}_\odot = -0.504\,858$, $-1.203\,679$, $-1.559\,933$, $-4.205\,334$ and $-4.890\,475$. At high stellar luminosities, neutrino emission leads to negative fractional luminosities. The effects of a finite derivative of the gravitational constant are clearly noticeable in curve 5 (see text for additional details).

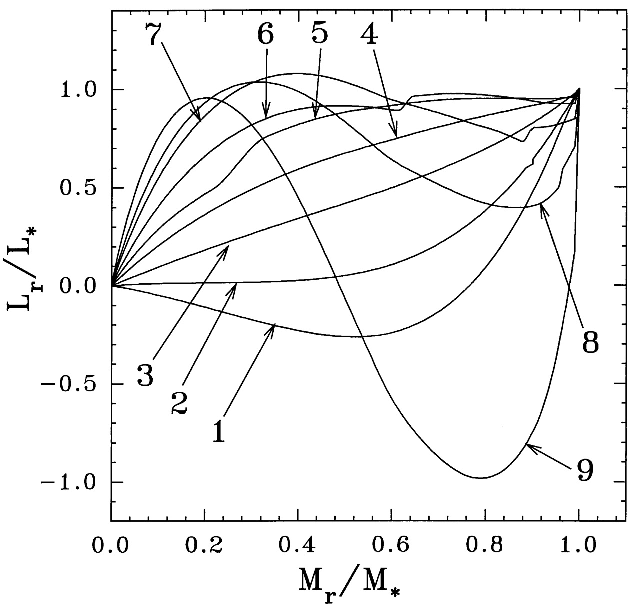

Figure 2. Same as Fig. 1, but for a 0.6-M$_\odot$ WD model. Notice the change in the vertical scale. The included evolutionary stages corresponding to $\log L/{\rm L}_\odot = -0.539\,606$, $-0.901\,381$, $-1.269\,676$, $-3.162\,055$, $-3.562\,184$, $-4.225\,259$, $-4.447\,918$, $-4.740\,054$ and $-4.971\,371$ are labelled from 1 to 9. Because of the larger stellar mass and the smaller radius, the effect of the running gravitational constant is much more noticeable in this case. This induces large negative luminosities (see text for additional details).

mass of the hydrogen and helium layers to be $M_{\rm H}/M_* = 10^{-5}$ and $M_{\rm He}/M_* = 0.01$, respectively. In order to present a detailed account of the effects of a non-zero $\dot{G}$ value upon the evolution of WDs, we shall refer in the text that follows to those models parametrized by

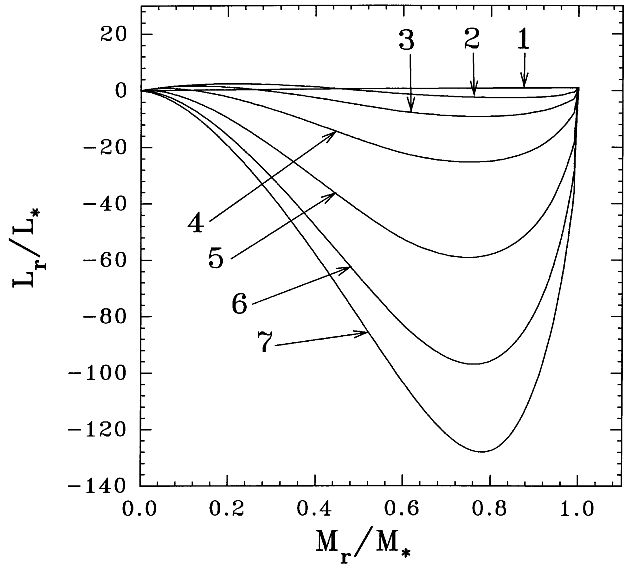

Figure 3. Same as Fig. 1, but for a 0.8-M$_\odot$ WD model. Notice the change in the vertical scale. Because of the scale, the bunch of curves labelled as 1 corresponds to early stages of evolution. Later evolutionary stages corresponding to $\log L/{\rm L}_\odot = -1.620\,706$, $-4.328\,015$, $-4.536\,697$, $-4.747\,211$, $-4.893\,521$, $-4.955\,040$ and $-4.976\,997$ are labelled as 2 to 7. Notice the very large negative luminosities at the last plotted evolutionary stage (see text for additional details).

$\omega = 600$, $t_{\rm U} = 12.5\,$Gyr and $t_{\rm B} = t_{\rm MS}$. This choice represents a rather extreme case in which the effects of $\dot{G}$ upon evolution are largely noticeable.

The first striking result is that, because of the presence of the last term in equation (19), the cooling of WDs is strongly accelerated, particularly at low luminosities. Owing to the fact that more massive WDs have smaller radii, this effect turns out to be far more dramatic in massive WDs.

We begin by examining Fig. 1, in which we show the profile of the internal luminosity of the WD relative to its surface value versus the fractional mass for a 0.4-M$_\odot$ WD model. In this figure, the curves labelled as 1 and 2 represent early WD evolutionary stages in which (as in the standard case) the main cooling agent is neutrino emission. In curve 3 neutrino contributions have faded away, and the change of slope in curve 4 at $M_r/M_* = 0.35$ is due to crystallization. Finally, the convexity in curve 5 at $M_r/M_* \gtrsim 0.40$ results from the effect of the variation in the value of G.

In Fig. 2 we show the same profile as the previous figure but now in the case of a 0.6-M$_\odot$ WD model. The first stages of evolution show a rather similar trend in the fractional luminosity, but for advanced ages the term including $\dot{G}$ completely takes over the evolution. Indeed, certain zones of the lowest luminosity model shown in the figure are characterized by a large negative luminosity, which indicates that such regions are absorbing energy from the central and outermost parts of the star. This effect is much more pronounced in the 0.8- and 1.0-M$_\odot$ models (see Figs 3 and 4). In these cases, at advanced stages of the cooling process, most of the stellar interior absorbs energy from the outermost layers. This occurs with negative relative luminosities that peak up to two orders of magnitude above the radiated luminosity. This effect is responsible for the strong acceleration of cooling at low luminosities.

In Fig. 5 we show the relation of central temperature ($T_{\rm c}$) as a function of log(L/L$_\odot$) for the 1-M$_\odot$ WD model for different values of ω. It is clearly noticeable that for $\dot{G} \neq 0$, models with

$-2.5 \gtrsim \log L/L_\odot \gtrsim -4.2$ show a larger slope the larger the value of $-\dot{G}$. At lower luminosities, the relationship abruptly becomes much steeper than in the standard case. The differences are also very noticeable in the value of T_c for a given value of $\log(L/L_\odot)$. This behaviour is intimately related to the existence of a large peak of negative luminosities at the stellar interior.

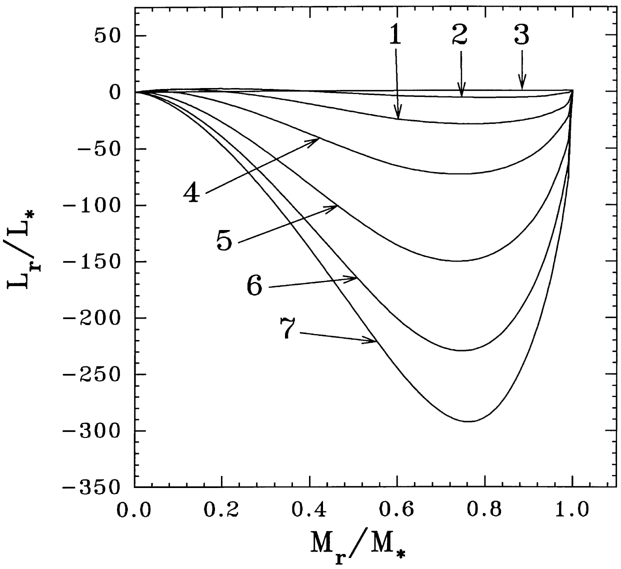

Figure 4. Same as Fig. 1, but for a 1.0-$M_\odot$ WD model. Notice the change in the vertical scale. Because of the scale, the bunch of curves labelled as 1 correspond to early stages of evolution. Later evolutionary stages corresponding to $\log L/L_\odot = -1.416\,044, -3.845\,380, -4.246\,737, -4.481\,437, -4.599\,128, -4.648\,833$ and $-4.668\,278$ are labelled as 2 to 7. Notice the very large negative luminosities at the last plotted evolutionary stage (see text for additional details).

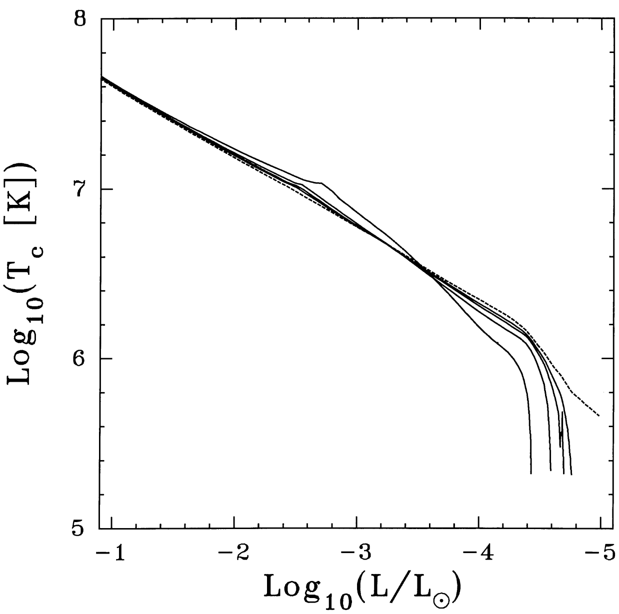

Figure 5. Logarithm of the central temperature in terms of the logarithm of the surface luminosity for the 1.0-$M_\odot$ WD model assuming $t_U = 12.5$ Gyr and $t_B = t_{MS}$ and the set of ω values: $\omega = 400$, 600, 800 and 1000. At low luminosities, curves are ordered from left to right for increasing ω values. For the sake of comparison, we show the relationship corresponding to the standard case of non-varying gravitational coupling constant (short dashed line). Note that at low luminosities, the value of the central temperature is significantly lower compared to the standard case, and its slope is several times larger.

It is particularly noteworthy to analyse the thermal profile of the WDs under these conditions. For this purpose, we show in Fig. 6 the logarithm of the internal temperature versus ξ [where $\xi = \log(1 - M_r/M_*)$] for the case of a 1.0-$M_\odot$ WD model. It is seen that the initial and intermediate states of evolution, in which the star can be considered as a genuine WD, are rather similar to previous results with a fixed G value. However, at the final stages calculated here, large differences appear. The most noticeable feature is that, despite the very small conductive opacity of the degenerate matter, the central parts of the star are *no longer isothermal*. Indeed, the internal temperature of the lowest luminosity model shown in Fig. 6 changes up to a factor 3 in 90 per cent of the internal mass of the star.

These results show that the evolution of WDs in the case of a running G value is markedly different from the standard case. It is very important to remark that these large differences are particularly noticeable at the range of luminosities at which the WDLF exhibits the sudden fall-off, which will be critical in the analysis of theoretically plausible results. Note also that the value of ω considered in the preceding results exceeds the upper bound implied by existing experiments (see Table 1).

In Figs 7–13 we show the relationship between the logarithm of the luminosity of the WD and its age for each mass value in the case of $t_B = t_{MS}$ and $t_U = 12.5$ Gyr and for $\omega = 400$, 600, 800, 1000, 2000 and 5000. This relation is fundamental in computing the theoretical WDLF, as will be discussed below. The effect of $\dot{G} \neq 0$ is clearly noticeable even in the case of the 0.4-$M_\odot$ WD model.

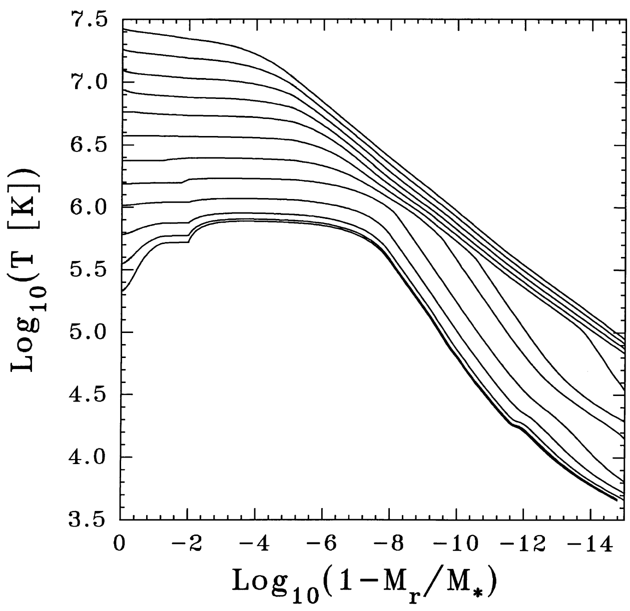

Figure 6. Logarithm of the internal temperature for a 1.0-$M_\odot$ WD model in terms of its external mass fraction for $\omega = 600, t_U = 12.5$ Gyr and $t_B = t_{MS}$. We depict evolutionary stages characterized by (from top to bottom) $\log L/L_\odot = -1.416\,044$, $-1.821\,964$, $-2.251\,024$, $-2.660\,958$, $-3.037\,227$, $-3.441\,015$, $-3.845\,380$, $-4.246\,737$, $-4.481\,437$, $-4.599\,128$, $-4.648\,833$ and $-4.668\,278$. The initial curves show a rather standard behaviour. However, as temperature decreases, it is clearly noticed that central parts of the star largely depart from the standard case, thus developing a large thermal gradient in highly degenerate layers, in clear contrast with the standard case.

While for high values of ω the function is rather similar to that of the standard case, the time spent by the WD in reaching $\log L/\mathrm{L}_\odot = -5$ falls to a half for $\omega = 1000$ and to a quarter when $\omega = 400$. Not surprisingly, for higher stellar mass values, these differences become dramatically larger. For example, for the 0.6-$\mathrm{M}_\odot$ WD object with $\omega = 1000$, the cooling time falls to a sixth of the standard one, whilst for $\omega = 400$ it falls by more than an order of magnitude. Finally, in the case of a 1.0-$\mathrm{M}_\odot$ WD model, if $\omega = 400$ the cooling time is almost *two orders of magnitude shorter* as compared with the standard case. These differences between the case $\dot{G} \neq 0$ and the standard one are large enough, even in the range of values of ω allowed by other experiments, that the consequences merit a careful analysis.

To compare our results with the observed WDLF of Leggett et al. (1998), we construct theoretical WDLFs from our DA WD evolutionary sequences according to equation (20). Note that the derived space densities are given in terms of intervals of bolometric magnitude M_{bol}. As mentioned in the foregoing section, all of the curves have been normalized to the observed space density of 3.39 stars per 10^3 pc^3. The results corresponding to the standard case of non-varying G are shown in Fig. 14 for assumed disc ages of 6–10 Gyr. Note that the best fit to the coolest WDs observed is obtained for disc ages of approximately 7–8 Gyr.

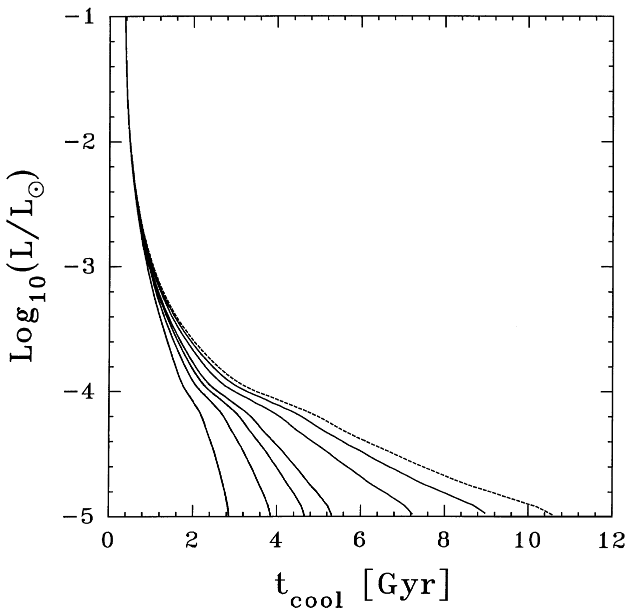

Figure 7. Logarithm of the luminosity versus age for a 0.4-$\mathrm{M}_\odot$ WD model assuming $t_{\mathrm{U}} = 12.5$ Gyr and $t_B = t_{\mathrm{MS}}$ for the set of ω values: $\omega = 400$, 600, 800, 1000, 2000 and 5000. Curves are ordered from left to right for increasing ω (decreasing $\dot{G}$) values. For the sake of comparison, in a short dashed line, we also show the standard cooling sequence. Note the acceleration of the cooling resulting from the change in the gravitational constant.

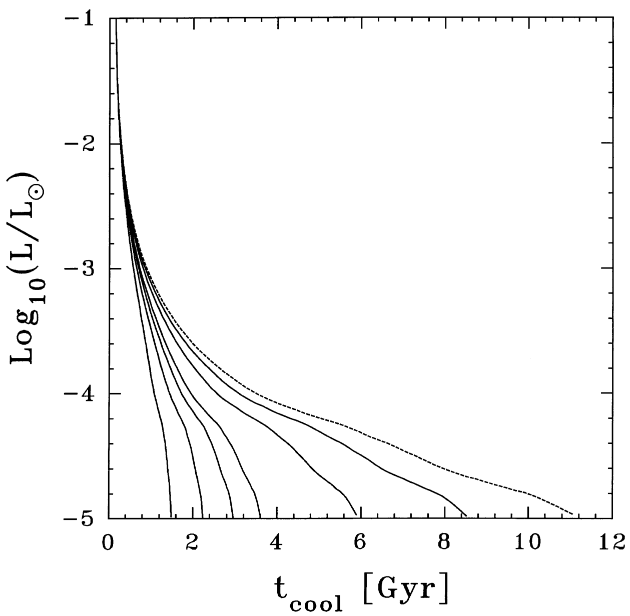

Figure 8. Same as Fig. 7, but for a 0.5-$\mathrm{M}_\odot$ WD model. Note that the acceleration is clearly larger as compared to the case of the 0.4-$\mathrm{M}_\odot$ WD model included in Fig. 7.

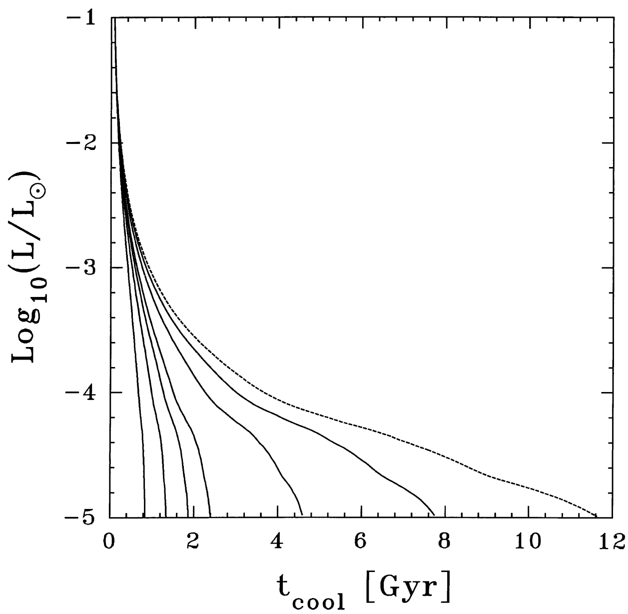

Figure 9. Same as Fig. 7, but for a 0.6-$\mathrm{M}_\odot$ WD model.

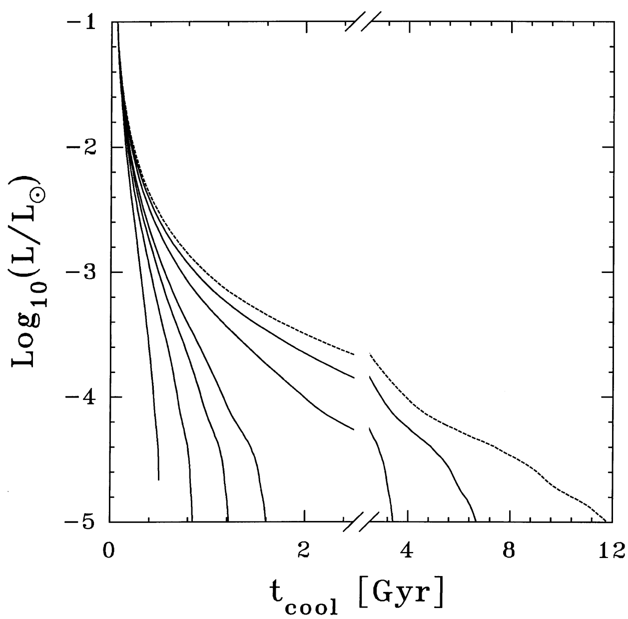

Figure 10. Same as Fig. 7, but for a 0.7-$\mathrm{M}_\odot$ WD model.

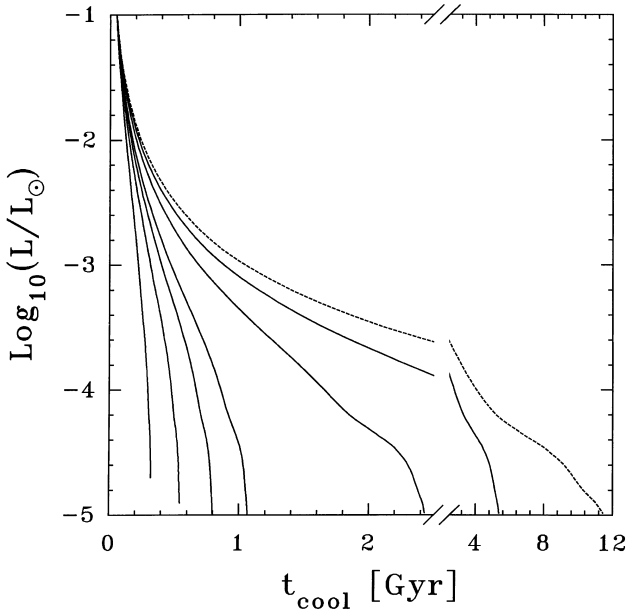

Figure 11. Same as Fig. 7, but for a 0.8-$M_{\odot}$ WD model.

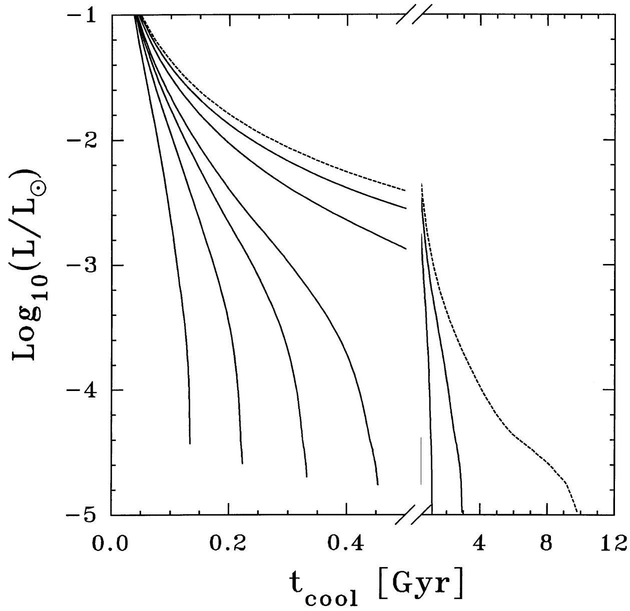

Figure 13. Same as Fig. 7, but for a 1.0-$M_{\odot}$ WD model.

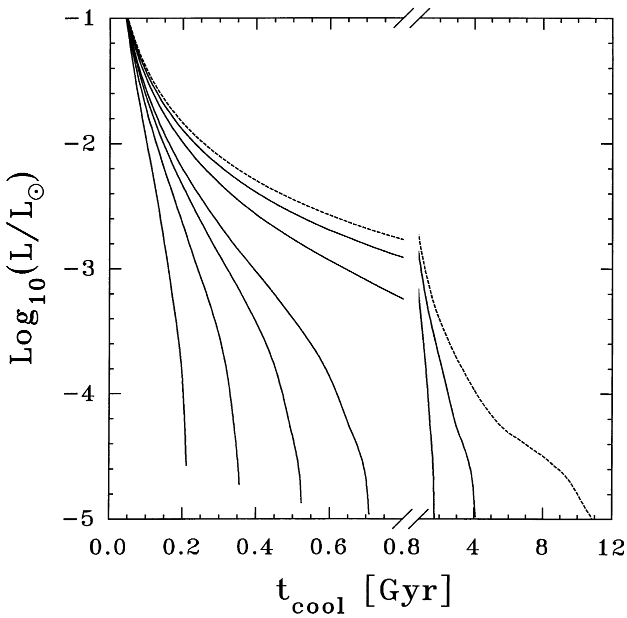

Figure 12. Same as Fig. 7, but for a 0.9-$M_{\odot}$ WD model.

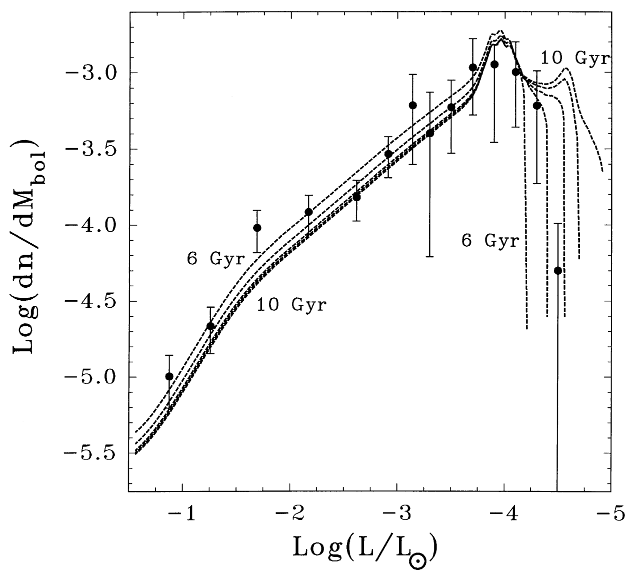

Figure 14. Theoretical WDLFs (dotted lines), depicted here per unit of bolometric magnitude, corresponding to our DA WD models for the standard case of non-varying gravitational coupling constant ($\dot{G} = 0$) and for assumed disc ages of 6–10 Gyr (at intervals of 1 Gyr). All of the curves, which are compared to the observational data of Leggett et al. (1998), have been normalized to the observed WD space density of 0.003 39 stars per cubic parsec. Note that the best fit to the dimmest WDs observed corresponds to assumed disc ages of approximately 7–8 Gyr.

The theoretical WDLFs resulting from the employment of a varying G are detailed in Figs 15–19 for various assumed disc ages of $T_{\rm disc} = 6, 7, 8, 9$ and 10 Gyr and $\omega = 400, 1000, 5000$ and 10 000. From these figures, which summarize the main results of the present paper, very interesting conclusions may be drawn. First, for very low values of ω, cooling is so fast that the resultant WDLF is almost *independent* of the assumed disc age. It is worthwhile to mention that, in this case, a final extinction of the WDLF is obtained but at lower luminosities than those predicted by observations. It is noticeable that, *even considering a value of* ω *as large as 10 000, appreciable differences are found in the theoretical WDLF.* For the range of values of disc ages considered in these figures, it is clear that values of $\omega < 5000$ should be discarded, as we find no agreement between observed and computed WDLFs. On the basis of these results, we conclude that the cooling of WDs represents a very powerful tool that allows one to constrain (or even to measure) the variation of the value of G with a far greater degree of sensitivity than that provided by existing experiments (see Table 1).

Another feature worthy of comment shown by our calculations in the context of a varying G is related to the mass of WDs at the faint regions of the WDLF. In this regard, we show in Figs 20–22 the minimum mass of the main-sequence progenitor contributing to the WDLF at a given luminosity. It is apparent from these figures that, in the case of a varying gravitational coupling constant corresponding to low ω values, progenitors with very low stellar masses will

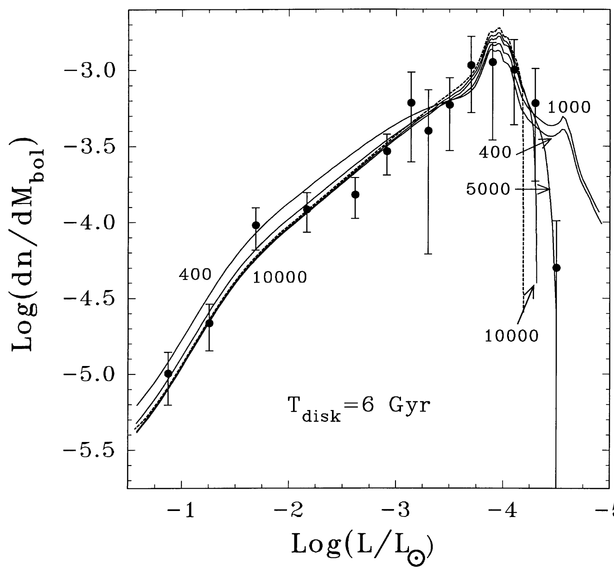

Figure 15. Theoretical WDLFs (depicted here per unit of bolometric magnitude) corresponding to our DA WD models with varying gravitational coupling constant and for the set of values: $\omega = 400$, 1000, 5000 and 10 000, assuming a disc age of $T_{\rm disc} = 6$ Gyr. At high luminosity, curves are ordered from top to bottom for increasing ω values. For the sake of comparison, we also depict the WDLF corresponding to $\dot{G} = 0$ (dotted line). The curves, which are compared to the observational data of Leggett et al. (1998), have all been normalized to the observed WD space density of 0.003 39 stars per cubic parsec. Note that the best fit to the dimmest WDs observed is given by the curve corresponding to $\omega = 5000$.

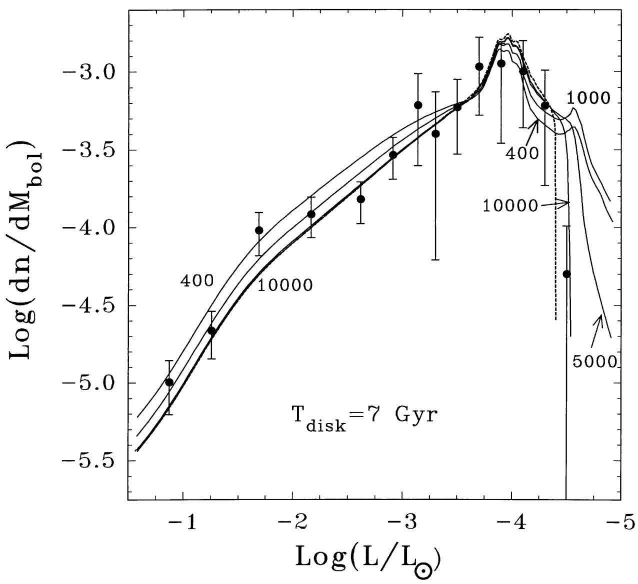

Figure 16. Same as Fig. 15, but for an assumed disc age of $T_{\rm disc} = 7$ Gyr. Note that the dimmest WDs observed can be fitted only by curves corresponding to $\omega > 5000$.

contribute to the WDLF even at luminosities near the observed turn-off luminosity, irrespective of the assumed disc age.

6 DISCUSSION AND CONCLUSIONS

We have analysed the evolution and cooling processes of white dwarf (WD) stars in the framework of a varying-G scenario. In doing so, we have tried to mantain a self-consistent framework: the particular form in which G varies is obtained from a cosmological solution of a Brans–Dicke theory of gravity. This stands for the case in which the star is aware of the cosmological evolution of the scalar that gives rise to the value of G. The other scenario, that of gravitational memory, was also investigated. In that case, the object remembers the value of Newton's coupling constant at the moment of star formation and conserves it during all its subsequent evolution. At the moment, there is no clear answer as to which of these scenarios happens in practice but it is natural to suppose and expect that some kind of evolution must occur; the characteristic time-scale of the object (the free-fall time $t_{\rm ff} \sim 1/\sqrt{G\rho}$) is in this case much smaller than the cosmological time-scale of G variation [$t_{\rm exp} \sim 1/H(t)$]. In addition, in other long-lived objects, like cosmic strings, this must be the case to avoid

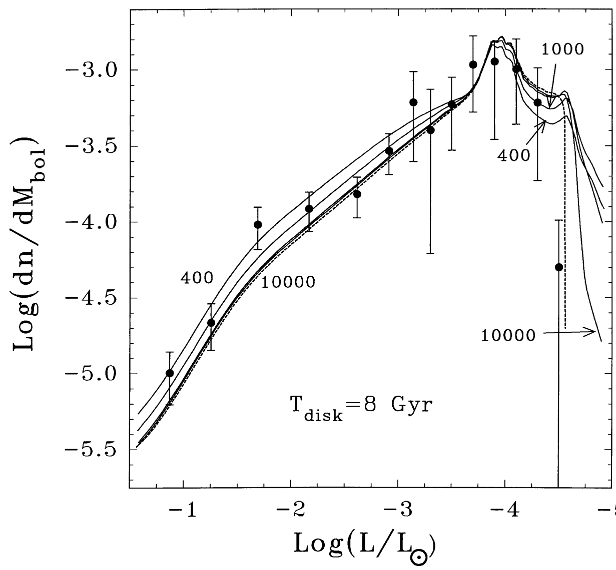

Figure 17. Same as Fig. 16, but for an assumed disc age of $T_{\rm disc} = 8$ Gyr. Note that the dimmest WDs observed can be fitted only by curves corresponding to $\omega > 10\,000$.

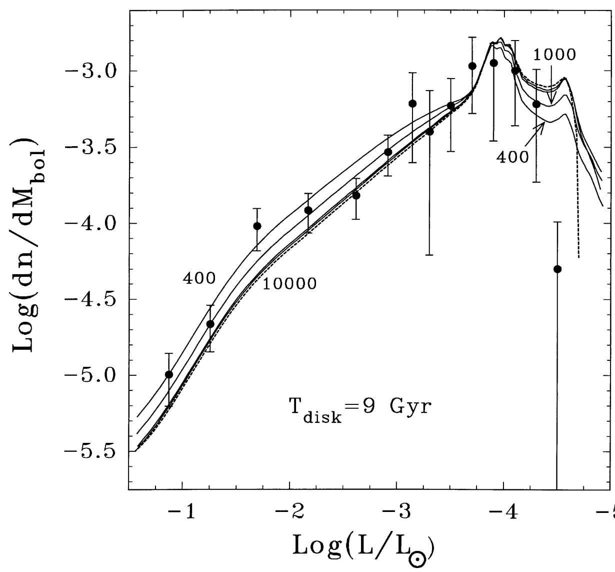

Figure 18. Same as Fig. 17, but for an assumed disc age of $T_{\rm disc} = 9$ Gyr. In this case, the dimmest WDs observed cannot be fitted by any of the curves shown in the figure.

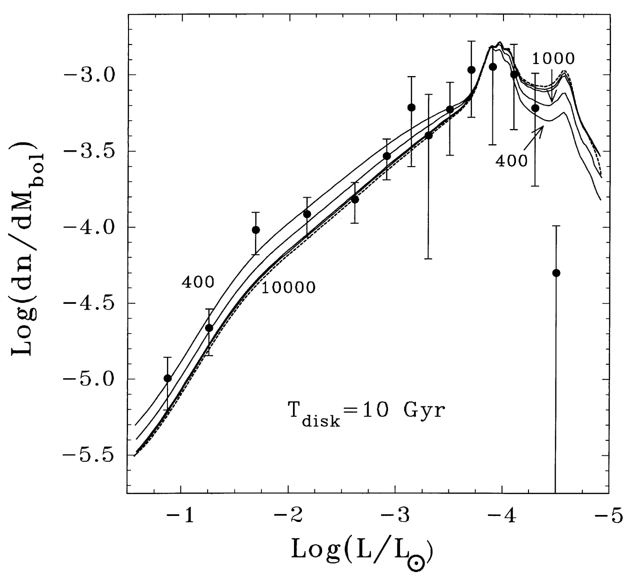

Figure 19. Same as Fig. 18, but for an assumed disc age of $T_{\rm disc} = 10$ Gyr.

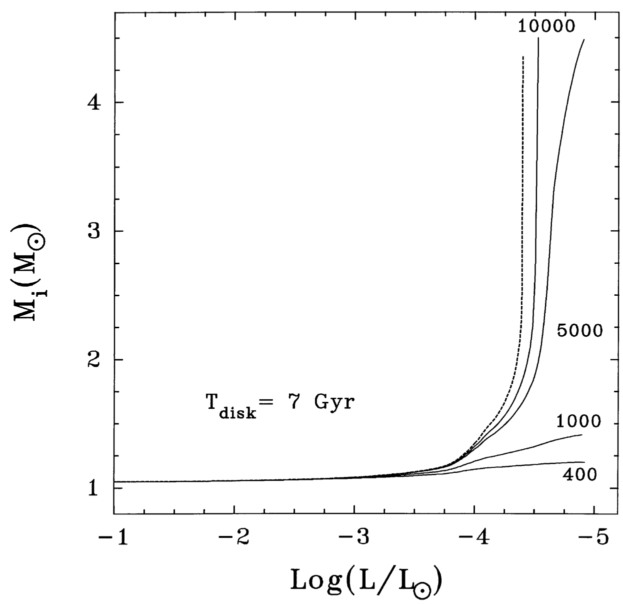

Figure 20. Minimum mass of the main-sequence progenitor contributing to the WDLF as a function of luminosity for a disc age of $T_{\rm disc} = 7$ Gyr. Solid lines correspond to the case of varying gravitational coupling constant for $\omega = 10\,000, 5000, 1000$ and 400 and dotted line to the case $\dot{G} = 0$. Note that in the case of varying gravitational coupling constant with low ω values, progenitors with very low stellar masses will contribute to the WDLF even at luminosities near the observed turn-off luminosity.

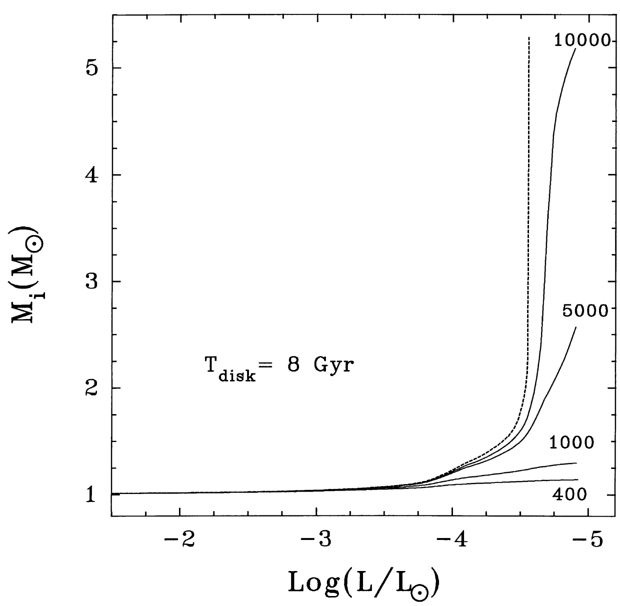

Figure 21. Same as Fig. 20, but for an assumed disc age of $T_{\rm disc} = 8$ Gyr.

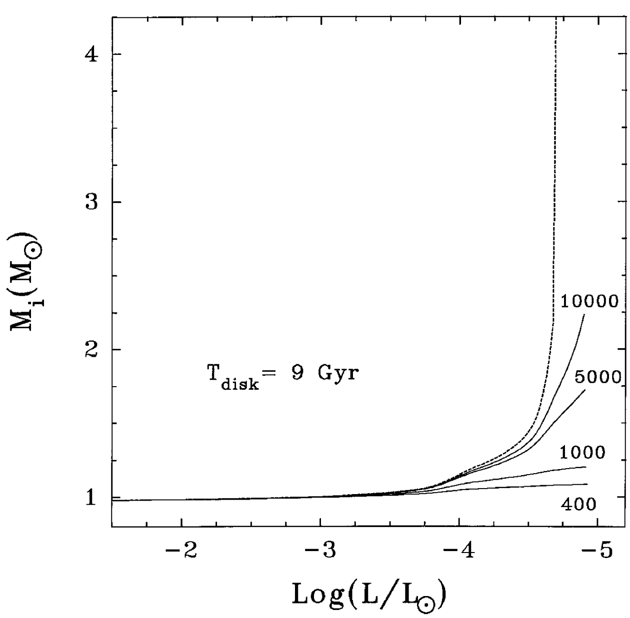

Figure 22. Same as Fig. 21, but for an assumed disc age of $T_{\rm disc} = 9$ Gyr. Now, even for the case of a very slow change of G, progenitors with very low stellar masses will contribute to the WDLF even at luminosities near the observed turn-off luminosity.

contradiction with observational results on the cosmic microwave background.

The gravitational treatment we followed is an approximation to a more complex situation: the exact value of G will depend not only on cosmological time but also on the space coordinates with centre in the star. This last dependence was overwritten in our case with that provided by the cosmological model.[3] This was also the case in the analysis of boson stars[4] (Torres 1997a; Torres et al. 1998a,b) and in the previous study on WDs, where an ad hoc dependence on $\dot{G}$ – which made $\dot{G}/G$ constant – was imposed (García-Berro et al. 1995). A fully relativistic treatment will surely introduce specific corrections, but as a result of the Newtonian nature of WDs, we expect that they will not be important.

The case of a fixed, bigger value of G, which is maintained during the whole lifetime of the star, would yield evolutionary curves and WD luminosity function (WDLF) diagrams close to, and hardly

[3] This is equivalent to thinking that the star is such a small perturbation of the background Friedmann universe as to disregard its influence in the cosmological solutions.

[4] There, the authors studied static stars at different times, with no evolution of the same configuration, contrary to what was done here.

distinguishable from, the standard case. This is not surprising because, if the effective value of $\dot{G}$ is zero, then the last term of equation (19) yields no contribution at all.

For a running value of G, we have found some striking results. The first of them is that the cooling of WDs is strongly accelerated, particularly for massive WDs at low luminosities, where the WDLF exhibits a sudden fall-off. More specifically, we found that, if the WD is able to feel the cosmological variation of G, even in the range of ω implied by other experiments, massive WDs cool down in a time up to two orders of magnitude faster than in the standard case. If ω has a low value, it causes massive WDs to develop large negative luminosities, far larger than the emitted ones. This indicates that the star absorbs energy from the outer layers in order to expand against gravity. Notably, in these cases, the interior strongly departs from an isothermal description in spite of the extremely low conductive opacities of the strongly degenerate plasma present in the deep interior. To our knowledge, there is no other case of strong departure from isothermality in degenerate layers in quasi-hydrostatic stellar evolution. For the case of large ω values, the differences are not so large, but are still clearly noticeable when compared to the standard case.

As a result of these differences, the theoretical WDLF turns out to be very sensitive to the value of the parameters ω and t_B. Thus, constructing theoretical WDLFs allows us to perform a detailed comparison with the observational WDLF, and then to deduce the range of parameters allowed by the evolution of WDs. As we have mentioned before, it seems to be the most sensitive test ever performed for theories of gravitation.

From the standard curves of the WDLF we can see that, if we adopt a $\dot{G}=0$ model, then the fall-off of the diagram entails an age for the Galactic disc between 7 and 8 Gyr (see also Leggett et al. 1998). Recall, on the other hand, that the currently favoured low values of the Hubble constant $H_0 \sim 65$–$80\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ yield estimates for the age of the Universe of order 7–9 Gyr, and so it is evident that the Galactic disc should be younger.

If we adopt now a $\dot{G}\neq 0$ model, we see that different ω values have different behaviours and that this also depends on the disc age. This age may be different from the Galactic age in the case of a later infall of massive material from a dark matter halo. To what extent the determination of the age of the Galactic disc (in situations where it would not depend on WD evolution) is independent of the $\dot{G}$ model is still a matter of study. Thus, to extract any conclusion we must assume a fixed age for the disc. We show several examples of WDLFs from 6 to 10 Gyr and different values of ω. In the first case, the lowest age analysed, $\dot{G}\neq 0$ models with extraordinarily low rate of variation such as those provided by a Brans - Dicke theory with $\omega = 5000$ seem to be a better fit to the data than the standard $\dot{G}=0$ case. For older discs ($\sim$10 Gyr), neither of the models (nor even the standard) fit the last troublesome point. Around the expected age, between 7 and 8 Gyr, a similar scheme to that of the lowest age happens, but now the variation in G is even more slow, and accordingly the Brans–Dicke parameter is even bigger. Considering the current observational status, the WDLF represents the strongest test of gravitational theories, providing a current $|\dot{G}/G|$ of order 10^{-14}, which is between one and three orders of magnitude more restrictive than previous bounds. It is amazing that the difference between the previous values of $\dot{G}$ and $\dot{G}\equiv 0$ is pretty well noticed in the diagrams, as well as its predictions and abilities in comparing with observations. In order to explore from where these differences in the WDLF diagrams arise, it is enough to study the minimum mass of the progenitor within each model. For instance, see the curves for $T_{\rm disc} = 7\,$Gyr. In the $\dot{G}=0$ case $M_{\rm i}(\mathrm{M}_\odot) \gg 4$, whilst for $\omega = 5000$, $M_{\rm i}(\mathrm{M}_\odot) > 2$. This, again, is a striking result from the gravitational point of view: any other test (weak or strong) is unable to see any deviation between general relativity and Brans–Dicke theory with such a huge parameter. To translate this in a practical test would entail and require one to develop an in-depth knowledge of the distribution of WDs at low luminosities in our neighbourhood.

It is worth noting, however, that the lowest luminosity point in the observed distribution is currently the least precisely determined: the number density in this region is extremely low and actually represents a handful of stars. It is still quite possible that this point (as well as other subsequent ones) may suffer substantive variations in its position in the near future. In fact, in a recent letter to *Nature* (Oswalt et al. 1996), it was located well above [log $(\mathrm{d}n/\mathrm{d}M_{\rm bol}) \sim -3$] the position we adopted in this work (Leggett et al. 1998). Although the observational analysis of Oswalt et al. (1996) has recently been criticized (Leggett et al. 1998), it is worth mentioning as an example of how a different position may affect our results and conclusions. We have studied the form of the WDLF following Ostwalt et al.'s observational points; this led us to conclude that, if the real WDLF was that of Ostwalt et al., any Brans–Dicke theory with values of ω bigger than 400 will be able to fit the observational data irrespective of the age of the Galactic disc. So different conclusions are the spin-off of a change in the position of the curves and thus it is important to clarify it.

In both situations, we have presented the case to transform the study of WDs in a powerful test of gravity. It now becomes obvious to study which changes are produced by different coupling functions in the gravitational sector of the action. This is important because theories that may deviate very much from general relativity at early times of the Universe may exist. For instance, power-law couplings are not constrained by weak-field tests and produce a cosmological logarithmic decrease in G, which is slower than the one explored here. How this affects the structure and how it differs from the Brans–Dicke model will be reported elsewhere.

It is worth mentioning that we have found that the simplest coupling possible, and values for it that make the theory almost indistinguishable from general relativity in any Solar system and strong tests, is enough to make evident very different stellar phenomena. This may be important in defining the low-energy form of the gravitational action: We have proven that astrophysics is sensitive to minimum (but conceptually important) differences in the underlying theory of gravity, and that this is compelling in deriving definite tests of the Universe.

A word of caution seems to be appropriate at this point. In this work we have not included a detailed treatment of crystallization processes in our evolving carbon–oxygen models, i.e. we have implicitly assumed that the mixture undergoes crystallization for any carbon–oxygen abundances. Nevertheless, in some works (see e.g. Stevenson 1980) it has been proposed that a crystallized carbon–oxygen mixture may prefer some defined relative abundances. If this occurs in nature and if the pre-WD evolution leads to an internal composition different from the preferred mixture, the crystallization process will be accompanied by a migration of part of the ions of the element present in excess. This would affect the gravitational energy release at such an evolutionary stage, and then it would lead to a sizeable modification of the WDLF, as found, for example, in Hernanz et al. (1994). We expect the inclusion of such a process to have some effect on the main conclusions of the present paper, although a quantitative exploration of the consequences can only be estimated by performing a calculation like the one we present here. Such a study is beyond the scope of the present paper.

Finally, we would like to comment why at present it is not possible to perform a similar analysis employing evolving neutron stars in place of WDs. As is well known, neutron stars have a much smaller radius and thus a much larger gravitational binding energy compared to WDs. Then, these objects seem to be even more promising for performing a study like the one we have presented. However, this is not the case, mainly for two reasons. First of all, from the theoretical point of view, it is well known that there is a large uncertainty in the actual equation of state of the matter of which neutron stars are made up. Consequently, at present we do not have models reliable enough for our purposes. Secondly, from the observational point of view, there is no equivalent 'neutron star luminosity function'. In fact, neutron stars are used to show the pulsar phenomenology. Thus, they are surrounded by an extended magnetosphere that masks the neutron star surface. This makes it hard to extract observational information on the evolutionary status of such objects.

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REFERENCES

Accetta F. S., Krauss L. M., Romanelli P., 1990, Phys. Lett. B, 248, 146
Alexander D. R., Ferguson J. W., 1994, ApJ, 437, 879
Althaus L. G., Benvenuto O. G., 1997, ApJ, 477, 313
Anchordoqui L. A., Perez Bergliaffa S., Torres D. F., 1997, Phys. Rev. D, 55, 5229
Anchordoqui L. A., Grunfeld A. G., Torres D. F., 1998, Grav. Cosmol. 4, 287
Anderson J. D., Keasey M. S. W., Lau E. L., Standish E. M., 1978, Acta Astron., 5, 43
Anderson J. D. et al., 1991, Proc. Astron. Soc. Aust., 9, 324
Barrow J. D., 1987, Phys. Rev. D, 35, 1805
Barrow J. D., 1992, Phys. Rev. D, 46, 3227
Barrow J. D., 1993, Phys. Rev. D, 47, 5329
Barrow J. D., 1994, Gen. Relativ. Gravitation, 26, 1
Barrow J. D., 1995, Phys. Rev. D, 51, 2729
Barrow J. D., Carr B. J., 1996, Phys. Rev. D, 54, 3920
Barrow J. D., Maeda K., 1992, Nucl. Phys. B, 341, 294
Barrow J. D., Mimoso J. P., 1994, Phys. Rev. D, 50, 3746
Barrow J. D., Parsons P., 1997, Phys. Rev. D, 55, 3906
Barrow J. D., Tipler F. J., 1986, The Anthropic Cosmological Principle. Oxford Univ. Press, Oxford
Bekenstein J. D., Sanders R. J., 1994, ApJ, 429, 480
Benvenuto O. G., Althaus L. G., 1997, MNRAS, 288, 1004
Benvenuto O. G., Althaus L. G., 1998, MNRAS, 293, 177
Bergmann P. G., 1968, Int. J. Theor. Phys., 11, 25
Berkin A. L., Hellings R. W., 1994, Phys. Rev. D, 49, 6442
Brans C., Dicke R. H., 1961, Phys. Rev., 24, 925
Bruckman W. F., Kazes E., 1977, Phys. Rev. D, 16, 261
Callan C. G., Friedan D., Martinec E. J., Perry M. J., 1985, Nucl. Phys. B, 262, 593
Campanelli M., Lousto C., 1993, Int. J. Mod. Phys. D, 2, 451
Casas J. A., García-Bellido J., Quirós M., 1992, Phys. Lett. B, 278, 94
Chandrasekhar S., 1937, Nat, 139, 757
Chin C.-W., Stothers R. B., 1976, Phys. Rev. Lett., 36, 833
Damour T., Pichon B., 1998, astro-ph 9807176
Damour T., Gibbons G. W., Taylor J. H., 1988, Phys. Rev. Lett., 61, 1151
D'Antona F., Mazzitelli I., 1978, A&A, 66, 453
D'Antona F., Mazzitelli I., 1989, ApJ, 347, 934
D'Antona F., Mazzitelli I., 1991, in Michaud G., Tutukov A., eds, Proc. IAU Symp. 145, Evolution of Stars: the Photospheric Abundance Connection. Kluwer, Dordrecht p. 339
Deaborn D. S., Schramm D. N., 1974, Nat, 247, 441
Degl'Innocenti S., Fiorentini G., Raffelt C. G., Ricci B., Weis A., 1996, A&A, 312, 345
Dickey J. O., Newhall X. X., Williams J. G., 1989, Adv. Space Res., 9, 75
Dirac P. A. M., 1937a, Nat, 139, 323
Dirac P. A. M., 1937b, Nat, 139, 1001
Domínguez-Tenreiro R., Yepes G., 1987, ApJ, 177, 5
Fontaine G., Brassard P., Wesemael F., Tassoul M., 1994, ApJ, 428, L61
Fradkin E. S., Tseytlin A. A., 1985, Nucl. Phys. B, 261, 1
García-Bellido J., Linde A., Linde D., 1994, Phys. Rev. D, 50, 730
García-Berro E., Hernanz M., Isern J., Mochkovitch R., 1988, Nat, 333, 644
García-Berro E., Hernanz M., Isern J., Mochkovitch R., 1995, MNRAS, 277, 801
Gurevich L. E., Finkelstein A. M., Ruban V. A., 1973, Ap&SS, 98, 101
Harada T., 1998, Phys. Rev. D, 57, 4802
Hawking S., 1972, Commun. Math. Phys., 25, 167
Hellings R. E., Adams P. J., Anderson J. D., Kessey M. S., Lau E. L., Standish E. M., Canuto V. M., Goldman I., 1983, Phys. Rev. Lett., 51, 18
Hellings R. E. et al., 1989, Int. J. Theor. Phys. 28, 1035
Hernanz M., García-Berro E., Isern J., Mochkovitch R., Segretain L., Chabrier G., 1994, ApJ, 434, 652
Hillebrandt W., Heintzmann H., 1974, Gen. Relativ. Gravitation, 5, 663
Holman R., Kolb E. W., Vadas S., Wang Y., 1991, Phys. Rev. D, 43, 3833
Iben I. Jr, Laughlin G., 1989, ApJ, 341, 312
Iben I. Jr, Tutukov A. V., 1984, ApJ, 282, 615
Iglesias C. A., Rogers F. J., 1993, ApJ, 412, 752
Kalligas D., Nortvedt K., Wagoner R. V., 1997, Phys. Rev. D, 56, 7627
Kang G., 1996, Phys. Rev. D, 54, 7483
Kaspi V. M., Taylor J. H., Ryba M., 1994, ApJ, 428, 713
Kim H., 1997, Nuovo Cimento B, 112, 329
Kippenhahn R., Weigert A., 1990, Stellar Structure and Evolution. Springer-Verlag, Berlin
Kippenhahn R., Weigert A., Hofmeister E., 1967, in Alder B., Fernbach S., Rottemberg M., eds, Methods in Computational Physics, Vol. 7. Academic Press, New York, p. 129
Kothari D. S., 1938, Nat, 142, 354
Krauss L. M., White M., 1992, ApJ, 394, 385
La D., Steinhardt P. J., 1989, Phys. Rev. Lett., 62, 376
Leggett S. K., Ruiz M. T., Bergeron P., 1998, ApJ, 497, 294
Liddle A. R., Mazumdar A., Barrow J. D., 1998, Phys. Rev. D, 58, 027302
Liebert J., Dahn C. C., Gresham M., Strittmatter P. A., 1979, ApJ, 233, 226
Liebert J., Dahn C. C., Monet D. G., 1988, ApJ, 332, 891
Lovelock C., 1985, Nucl. Phys. B, 273, 413
Mathiazhagen C., Johri V. B., 1984, Classical Quantum Gravity, 1, L29
Matsuda T., Nariai H., 1973, Prog. Theor. Phys., 49, 1195
Mimoso J. P., Wands D., 1995, Phys. Rev. D, 51, 477
Morrison L. V., 1973, Nat, 241, 519
Nandi K. K., Bhattacharjee B., Alam S. M. K., Evans J., 1998, Phys. Rev. D, 57, 823
Nariai H., 1969, Prog. Theor. Phys., 42, 544
Noh H.-R., Scalo J., 1990, ApJ, 352, 605
Nordvedt K., 1968, Phys. Rev. 169, 1017
Novak J., 1998a, Phys. Rev. D, 57, 4789
Novak J., 1998b, Phys. Rev. D, 58, 064019
Oswalt T. D. , Smith J. A., Wood M. A., Hintzen P., 1996, Nat, 382, 692
Rainer M., Zhuk A., 1996, Phys. Rev. D, 54, 6186
Reasenberg R. D., 1983, Phil. Trans. R. Soc. Lond. A, 310, 227
Reasenberg R. D., Shapiro I. I., 1976, in Sanders J. H., Wapstra A. H., eds, Atomic Masses and Fundamental Constants, Vol. 5. Plenum, New York
Reasenberg R. D., Shapiro I. I., 1978, in Halpern L., ed., On the Measurement of Cosmological Variations of the Gravitational Constant. University Presses of Florida, Gainesville

Reasenberg R. D. et al., 1979, ApJ, 234, L219
Salmona A., 1967, Phys. Rev., 154, 218
Saumon D., Chabrier G., Van Horn H. M., 1995, ApJS, 99, 713
Schmidt M., 1959, ApJ, 129, 243
Shapiro I. I., 1990, in Ashby N., Bartlett D., Wyss W., eds, General Relativity and Gravitation 1989. Cambridge Univ. Press, Cambridge
Shapiro I. I., Ash M. E., Ingalls R. P., Smith W. B., Campbell D. B., Dyce R. B., Jurgens R. F., Rettengill G. H., 1971, Phys. Rev. Lett., 26, 1132
Sisterna P., Vucetich H., 1990a, Phys. Rev. D, 41, 1034
Sisterna P., Vucetich H., 1990b, Phys. Rev. Lett., 72, 454
Steinhardt P. J., Accetta F. S., 1990, Phys. Rev. Lett., 64, 2740
Stevenson D. J., 1980 J. Phys. Suppl., C2-53
Teller E., 1948, Phys. Rev., 73, 801
Thorne K. S., Dykla J. J., 1971, ApJ, 166, L35
Thorsett S. E., 1996, Phys. Rev. Lett., 77, 1432
Torres D. F., 1995, Phys. Lett. B, 359, 249
Torres D. F., 1997a, Phys. Rev. D, 56, 3478
Torres D. F., 1997b, Phys. Lett. A, 225, 13
Torres D. F., Helmi A., 1996, Phys. Rev. D, 54, 6181
Torres D. F., Vucetich H., 1996, Phys. Rev. D, 54, 7373
Torres D. F., Liddle A. R., Schunck F. E., 1998a, Phys. Rev. D, 57, 4821
Torres D. F., Schunck F. E., Liddle A. R., 1998b, Classical Quantum Gravity, 15, 3701
Vila S. C., 1976, ApJ, 206, 213
Vucetich H., 1996, Bol. Acad. Nac. Córdoba, 61, 1
Webb J. K., Flambaum V. V., Churchill C. W., Drinkwater M. J., Barrow J. D., 1999, Phys. Rev. Lett., 82, 884
Will C. M., 1993, Theory and Experiment in Gravitational Physics, 2nd edn. Cambridge Univ. Press, Cambridge
Will C. M., 1998, gr-qc 9811036
Williams J. G., Sinclair W. S., Yoder C. F., 1978, Geophys. Res. Lett., 5, 943
Winget D. E. et al., 1987, ApJ, 315, L77
Wood M. A., 1992, ApJ, 386, 539
Wood M. A., 1995, in Koester D., Werner K., eds, White Dwarfs. Springer-Verlag, Berlin, p. 41
Wood M. A., Oswalt T. D., 1998, ApJ, 497, 870
Yuan J. W., 1989, A&A, 224, 108

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