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Evolution of DA white dwarfs in the context of a new theory of convection

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ABSTRACT

In this study we compute the structure and evolution of carbon–oxygen DA (hydrogen-rich envelope) white dwarf models by means of a detailed and updated evolutionary code. We consider models with masses from 0.5 to 1.0 $M_\odot$ and we vary the hydrogen layer mass in the interval $10^{-13} \leq M_H/M \leq 10^{-4}$. In particular, we treat the energy transport by convection within the formalism of the full-spectrum turbulence theory, as given by the Canuto, Goldman & Mazzitelli (CGM) model. We explore the effect of various hydrogen layer masses on both the surface gravity and the hydrogen burning. Convective mixing at low luminosities is also considered.

One of our main interests in this work has been to study the evolution of ZZ Ceti models, with the aim of comparing the CGM and mixing-length theory (MLT) predictions. In this connection, we find that the temperature profile given by the CGM model is markedly different from that of the ML1 and ML2 versions of the MLT. In addition, the evolving outer convection zone behaves differently in both theories.

We have also computed approximate effective temperatures for the theoretical blue edge of the DA instability strip by using thermal time-scale arguments for our evolving DA models. In this context, we found that the CGM theory leads to blue edges that are cooler than the observed ones. However, because the determination of atmospheric parameters of ZZ Ceti stars is dependent on the assumed convection description in model atmosphere calculations, observed blue edges based on model atmospheres computed considering the CGM theory are required in order to perform a self-consistent comparison of our results with observations. Finally, detailed non-adiabatic pulsational computations of ZZ Ceti models considering the CGM convection would be necessary to place the results found in this paper on a firmer basis.

Key words: convection – stars: evolution – stars: variables: other – white dwarfs.

1 INTRODUCTION

White dwarf stars with hydrogen lines (spectral type DA) constitute the most numerous group of observed white dwarfs. Since the first DA white dwarf was reported to exhibit multiperiodic luminosity variations (see McGraw 1979), the interest in these objects has greatly increased. Rapid progress in this field has been possible thanks to the development of powerful theoretical tools as well as an increasing degree of sophistication in observational techniques. Over the years, various studies have presented strong evidence that pulsating DA white dwarfs (DAV) or ZZ Ceti stars represent an evolutionary stage in the cooling history of the majority of, if not all, DA white dwarfs (Weidemann & Koester 1984 and references therein).

Studies carried out, notably by Dolez & Vauclair (1981) and Winget et al. (1982), on the basis of detailed non-radial,

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non-adiabatic pulsation calculations, demonstrated that the κ-mechanism operating in the hydrogen partial ionization zone is responsible for the g(gravity)-mode instabilities in ZZ Ceti stars. From then on, pulsating white dwarfs have captured the interest of numerous investigators, who have employed the powerful technique of white dwarf seismology to derive fundamental parameters of these stars, such as the stellar mass, chemical composition and stratification of the outer layers (see, for example, Winget et al. 1994 and Bradley 1996 in the context of DB and DA, respectively). In particular, pulsation calculations have shown that the theoretical determination of the location of the blue (hot) edge of the instability strip (which marks the onset of pulsations) in the Hertzsprung–Russell diagram is mostly sensitive to the treatment of convection in envelope models. Accordingly, a comparison with the location of the hottest observed ZZ Ceti stars allows us to obtain valuable information about the efficiency of convection in such stars (Winget et al. 1982; Tassoul, Fontaine & Winget 1990; Bradley & Winget 1994; Bradley 1996 and references cited therein).

Unfortunately, the determinations of atmospheric parameters of ZZ Ceti stars is strongly dependent on the assumed convection description in model atmosphere calculations, against which the observed spectrum is compared. The theoretical analysis carried out by Bergeron, Wesemael & Fontaine (1992a) is particularly noteworthy in this regard. In fact, these authors demonstrated that the predicted emergent fluxes, colour indices and equivalent widths of DA white dwarfs, notably of the ZZ Ceti stars, are very sensitive to the details of the parametrization of the mixing-length theory (MLT) of convection. In particular, Bergeron et al. (1992a) concluded that the uncertainties in the observational definition of the blue edge of the ZZ Ceti instability strip brought about by different convective efficiencies may be appreciable, irrespective of the observational technique that is used.

Needless to say, the employment of MLT to deal with convection represents a serious shortcoming in theoretical studies of white dwarfs. In particular, MLT approximates the whole spectrum of eddies necessary to describe the stellar interiors by a single large eddy. This representation introduces in the description of the model certain free parameters (Böhm-Vitense 1958), which, in most stellar studies, are reduced to a single one: the distance l travelled by the eddy before thermalizing with the surrounding medium. l is written as $l=\alpha H_p$ where H_p is the pressure scaleheight and α is a free parameter. Obviously, the existence of such an adjustable parameter prevents stellar evolution calculations from being completely predictive. In white dwarf studies, several parametrizations of the MLT are extensively employed. Specifically, the ML1, ML2 and ML3 versions, which differ in the choice of coefficients appearing in the expressions of the convective flux, average velocity and convective efficiency (see Tassoul et al. 1990 for details) are associated, respectively, with increasing convective efficiency (for the ML1 and ML2 versions $\alpha=1$, while ML3 is the same as ML2 but with $\alpha=2$).

Over the last two decades, an unparalleled effort has been devoted to the observational determination of the location of the blue edge of the instability strip of the ZZ Ceti stars (see Wesemael et al. 1991 for a review). Based on different observational techniques, the majority of the studies cited by Wesemael et al. point towards a temperature of the blue edge, defined by the ZZ Ceti star G117-B15A, of approximately 13 000 K. New observational data and improved model atmospheres tend to indicate a significantly lower temperature for the blue edge. In fact, from ultraviolet observations performed with the *Hubble Space Telescope*, Koester, Allard & Vauclair (1994) have shown that model atmospheres calculated with the ML1 parametrization do not fit satisfactorily to the ultraviolet spectrum of G117-B15A. Instead, a more efficient convection, as given by the ML2 version, is required to explain the observed spectrum. On the basis of this result, Koester et al. derived for G117-B15A an effective temperature of $12\,250 \pm 125$ K. In view of the fact that the ultraviolet spectra of other candidates for the blue edge of the ZZ Ceti instability strip are similar to that of G117-B15A, Koester et al. concluded that the blue edge temperature is likely to be around 12 250 K, a value which is considerably lower than found in previous studies.

In a still more recent analysis, Bergeron et al. (1995) inferred an even lower effective temperature for G117-B15A. More precisely, on the basis of new optical and ultraviolet analyses of ZZ Ceti stars, Bergeron et al. succeeded in constraining the convective efficiency used in model atmosphere calculations of these stars. Indeed, they demonstrated that a parametrization less efficient than ML2, particularly the ML2/$\alpha=0.6$ version (intermediate in efficiency between ML1 and ML2), yields ultraviolet temperatures that are completely consistent with the optical determinations. In this respect, they derived for G117-B15A an effective temperature of 11 620 K. In the sample analysed by Bergeron et al., the blue edge of the instability strip is defined by the star G226-29, for which they obtained a temperature of 12 460 K and a surface gravity of $\log g=8.29$. Finally, Fontaine et al. (1996) presented a reanalysis of the observed pulsation modes of G117-B15A that reinforces the conclusions of Bergeron et al. (1995) on the atmospheric parameters of this variable white dwarf. Fontaine et al. concluded that a model atmosphere of G117-B15A characterized by an effective temperature of $\approx 11\,500$ K and ML2/$\alpha=0.6$ convection is consistent with optical and ultraviolet spectroscopic observations as well as with the observed pulsation amplitudes in different wavelength bands.

Another conclusion drawn by Bergeron et al. (1995) (see also Wesemael et al. 1991) worthy of comment is related to the fact that convective efficiency in the upper atmosphere appears to be quite different from that in the deeper envelope. In fact, according to non-adiabatic pulsation calculations (Bradley & Winget 1994 and references cited therein), a more efficient convection than that provided by ML2/$\alpha=0.6$ is needed in the deeper envelope to explain the observed DA blue edge. This result, which reflects the inability of MLT to describe convection throughout the entire outer convection zone adequately, has indeed been borne out by recent detailed hydrodynamical simulations of convection in ZZ Ceti stars (see later in this section). Other objections have been raised against MLT. For instance, the value of α, which is usually derived from solar radius adjustments, is larger than unity, in contrast to the basic postulates of MLT. In addition to this, the procedure of applying the solar α parameter to other stars is not entirely satisfactory, at least in the context of red giant stars, for which a wide range of α values is needed (Stothers & Chin 1995, 1997). Finally, MLT does not fit laboratory convection data (Canuto 1996).

Fortunately, a considerable effort has been devoted in recent years to improving some of the basic postulates of MLT. In particular, Canuto & Mazzitelli (1991, 1992, hereafter CM) and more recently Canuto, Goldman & Mazzitelli (1996, hereafter CGM) developed two convection models based on the full-spectrum turbulence (FST) theory, which represent a substantial progress in modelling stellar convection (we refer the reader to Canuto & Christensen-Dalsgaard 1997 for a recent review on local and non-local convection models). Both of the treatments resolve essentially two main shortcomings of the MLT.

(i) The MLT one-eddy approach is replaced by a full spectrum of turbulent eddies derived on the basis of amply tested, modern theories of turbulence. As discussed by CM, the one-eddy approximation is only justified for viscous fluids and becomes completely inadequate for almost-inviscid stellar interiors.

(ii) There are no adjustable parameters.

These features render the CM and CGM models substantially more solid and complete than MLT. The CM model in particular has successfully passed a wide variety of laboratory and astrophysical tests, performing in all cases much better than MLT. With regard to the astrophysical case, the CM model has been applied to the study of different kinds of stars and evolutionary phases. Applications include the solar model (see CM), the effective temperature of which is fitted within ≈ 0.5 per cent accuracy, red giants (D'Antona, Mazzitelli & Gratton 1992 and Stothers & Chin 1995, 1997), for which MLT may require various values of α for different stars, helioseismology (Paternò et al. 1993 and Monteiro, Christensen-Dalsgaard & Thompson 1996 amongst others), low-mass stars (D'Antona & Mazzitelli 1994) and stellar atmospheres (Kupka 1996). In the white dwarf context, the CM model has been shown to be a very valuable tool as well. In fact, Althaus & Benvenuto (1996) (see also Mazzitelli & D'Antona 1991) studied the evolution of carbon–oxygen white dwarfs with helium-rich envelopes (spectral type DB) and found that the CM model predicts theoretical blue edges for the DB instability strip in good agreement with the observations of Thejll, Vennes & Shipman (1991). Mazzitelli (1995) has also applied the CM model to the study of convection in DA white dwarfs. More recently, Althaus & Benvenuto (1997a) used the CM model to study the evolution of helium white dwarfs of low and intermediate masses.

CGM have recently improved the CM model by developing a self-consistent model for stellar turbulent convection, which, like the CM model, includes the full spectrum of eddies but computes the rate of energy input self-consistently. More precisely, CGM resolve analytically a full-turbulence model in the local limit, taking into account the fact that the energy input from the source (buoyancy) into the turbulence depends on both the source and the turbulence itself. At low and intermediate convective efficiencies, the CGM model gives rise to higher convective fluxes than those given by the CM model. CGM find that the main-sequence evolution of a solar model does not differ appreciably from the CM predictions but demands a smaller overshooting to fit the solar radius, in agreement with new observational data. D'Antona, Caloi & Mazzitelli (1997) have also applied the CGM model to study the problem of globular cluster ages. In the white dwarf domain, the CGM model also fares better than the CM model (Althaus & Benvenuto 1997b and Benvenuto & Althaus 1997). Finally, Benvenuto & Althaus (1998) have applied the CGM model to study the evolution of low- and intermediate-mass helium white dwarfs with hydrogen envelopes.

In order to assess to what extent the MLT is able to provide a reliable description of convection in the outer layers of DA white dwarfs, Ludwig, Jordan & Steffen (1994) have performed detailed two-dimensional hydrodynamical simulations of convection at the surface of a ZZ Ceti star. They demonstrated that the temperature profile in the outer layers of a $T_{\rm eff}=12\,600$ K, $\log g=8.0$ DA model cannot be reproduced with model atmospheres calculated with MLT and with a single value of α. Instead, the value of α is to be varied from 1.5 in the upper atmosphere to 4 in much deeper layers, in order for the hydrodynamical temperature profile to be reasonably well represented. Another result obtained by Ludwig et al. is that their hydrodynamical simulations lead to convective zones that are too shallow for the κ, γ mechanism to give rise to unstable g modes, implying a blue edge for the ZZ Ceti instability strip that is significantly cooler than 12 600 K. This important finding has recently been borne out by Gautschy, Ludwig & Freytag (1996) on the basis of non-radial, non-adiabatic pulsation calculations. Gautschy et al. included in their analysis Ludwig et al.'s hydrodynamical simulations to derive the structure of the convective layers of their white dwarf models. This feature represents substantial progress compared with earlier investigations, which employ MLT to deal with convection. Gautschy et al. are unable to find a theoretical blue edge for the ZZ Ceti instability strip consistent with the observations of Bergeron et al. (1995) and conclude that, according to their calculations, it lies between 11 400 K and 11 800 K (for a model with $\log g=8$).

The present study is devoted to presenting new calculations of the evolution of carbon–oxygen DA white dwarfs, considering the model for stellar turbulent convection of CGM as well as the various parametrizations of MLT commonly used in the study of these stars. Attention is focused mainly on those evolutionary stages corresponding to the ZZ Ceti effective temperature range. The calculations are carried out with a detailed white dwarf evolutionary code in which we include updated equations of state, opacities and neutrino emission rates. Furthermore, we take into account crystallization, convective mixing and hydrogen burning. To explore the sensitivity of the results to various input model parameters, we vary both the model mass from 0.50–1.0 $M_\odot$, which covers most of the observed mass distribution of DA white dwarfs (Bergeron, Saffer & Liebert 1992b; Marsh et al. 1997), and the hydrogen layer mass $M_{\rm H}$ in the interval $10^{-13} \le M_{\rm H}/M \le 10^{-4}$, where M is the model mass. Finally, the hydrogen–helium transition zone is assumed to be almost discontinuous. Details of our evolutionary code and its main physical constituents are presented in Section 2. Section 3 is devoted to analysing our results and, finally, in Section 4 we summarize our findings.

2 COMPUTATIONAL DETAILS AND INPUT PHYSICS

The white dwarf evolutionary code we employed in this study was developed by us independently of other researchers and it has been used to study different problems connected to white dwarf evolution as well as the cooling of the so-called strange dwarf stars (Benvenuto & Althaus 1996). Details of the code and its constitutive physics can be found in Althaus & Benvenuto (1997a) and Benvenuto & Althaus (1997, 1998). In what follows we restrict ourselves to a few brief comments.

The equation of state for the low-density regime is that of Saumon, Chabrier & Van Horn (1995) for hydrogen and helium plasmas. The treatment for the high-density, completely ionized regime appropriate for the white dwarf interior includes ionic and photon contributions, coulomb interactions, partially degenerate electrons and electron exchange and Thomas–Fermi contributions at finite temperature. Radiative opacities for the high-temperature regime ($T \geq 8000$ K) are those of OPAL (Iglesias & Rogers 1993) with metallicity $Z=0.001$, whilst for lower temperatures we use the Alexander & Ferguson (1994) molecular opacities. Molecular effects become relevant at temperatures as high as 5000 K and below 2500 K they are dominant (Alexander & Ferguson 1994). We shall see that convective mixing at low effective temperatures may in some cases change the outer layer composition from almost pure hydrogen to almost pure helium. This being the case, we had to rely, in the low-temperature regime, on the older tabulation of Cox & Stewart (1970) for helium composition. Conductive opacities for the low-density regime are taken from Hubbard & Lampe (1969) as fitted by Fontaine & Van Horn (1976). Conductive opacities for the liquid and cristalline phases, and the various mechanisms of neutrino emission relevant to white dwarf interiors are taken from the works of Itoh and collaborators (see Althaus & Benvenuto 1997a and references cited therein). It is worth mentioning that neutrino coding is dominant during the hot phases of white dwarf evolution but they become completely negligible at the ZZ Ceti effective temperature range.

The procedure we followed to generate the initial models of different masses is described in Benvenuto & Althaus (1995). Such initial models were constructed starting from a 0.55-$M_\odot$ carbon–oxygen white dwarf model kindly provided to us by Professor Francesca D'Antona. We assumed the same core chemical composition (consisting of a mixture of carbon and oxygen, see fig. 1 of Benvenuto & Althaus 1995) for all of them, notwithstanding the changes in the internal chemical composition that are expected to occur for models with different masses because of differences in the pre-white dwarf evolution of progenitor objects. The carbon–oxygen core of our models is surrounded by an almost pure helium envelope, the mass of which ($M_{\rm He}$) was varied in the range $10^{-6} \leq M_{\rm He}/M \leq 10^{-2}$. In DA white dwarfs there is an almost pure hydrogen envelope on top of the helium layers. Unfortunately, the mass of this hydrogen envelope is only weakly constrained by pre-white dwarf evolutionary calculations (D'Antona & Mazzitelli 1991) and by non-adiabatic pulsation studies (Fontaine et al. 1994). In recent years, however, strong evidence has been accumulated in favour of the idea that some ZZ Ceti stars appear to have thick hydrogen layers (Fontaine et al. 1994). In the present study, we decided to treat the mass of the hydrogen envelope as basically a free parameter within the range $10^{-13} \leq M_{\rm H}/M \leq 10^{-4}$. We refer the reader to Benvenuto & Althaus (1998) for details about the procedure we follow to add a hydrogen envelope at the top of our models. We want to mention that we assumed the hydrogen/helium transition zone to be almost discontinuous.

We also included in our evolutionary calculations the release of latent heat during crystallization (see Benvenuto & Althaus 1997) and the effect of convective mixing. We shall see in particular that the onset of crystallization in our more massive models occurs at effective temperatures close to the observed blue edge. Finally, we also considered the effects of hydrogen burning by including in our numerical code the complete network of thermonuclear reaction rates for hydrogen burning corresponding to the proton–proton chain and the CNO bi-cycle. Nuclear reaction rates are taken from Caughlan & Fowler (1988) and β decay rates from Wagoner (1969), taking into account the corrections for their Q values resulting from the effect of neutrino losses. Electron screening is from Wallace, Woosley & Weaver (1982). We consider the following chemical species: ^{1}H, ^{2}H, ^{3}He, ^{4}He, ^{7}Li, ^{7}Be, ^{8}B, ^{12}C, ^{13}C, ^{13}N, ^{14}N, ^{15}N, ^{15}O, ^{16}O, ^{17}O and ^{17}F.

The most relevant feature of the present study is the employment of FST theory to deal with energy transport by convection. This theory represents a substantial improvement over MLT, particularly in the treatment of low-viscosity fluids like the ones present in in stellar interiors, for which the MLT one-eddy assumption is completely inadequate. In particular, a self-consistent model without adjustable parameters based on the FST approach has been recently formulated by CGM. These authors fitted their theoretical results for the convective flux $F_{\rm c}$ with the expression

$$F_{\rm c}=KTH_{\rm p}^{-1}(\nabla-\nabla_{\rm ad})\Phi, \quad (1)$$

where $K=4acT^3/3\kappa\rho$ is the radiative conductivity, $H_{\rm p}$ is the pressure scaleheight and ∇ and $\nabla_{\rm ad}$ are, respectively, the true and adiabatic temperature gradients. Φ is given by

$$\Phi=F_1(S)F_2(S), \quad (2)$$

where

$$F_1(S)=\left(\frac{Ko}{1.5}\right)^3 aS^k[(1+bS)^l-1]^q, \quad (3)$$

and

$$F_2(S)=1+\frac{cS^{0.72}}{1+dS^{0.92}}+\frac{eS^{1.2}}{1+fS^{1.5}}. \quad (4)$$

Here, Ko is the Kolmogorov constant (assumed to be 1.8), $S=162A^2(\nabla-\nabla_{\rm ad})$, and the coefficients are given by $a=10.8654$, $b=4.89073\times10^{-3}$, $k=0.149888$, $l=0.189238$, $q=1.85011$, $c=1.08071\times10^{-2}$, $d=3.01208\times10^{-3}$, $e=3.34441\times10^{-4}$ and $f=1.25\times10^{-4}$. A is given by

$$A=\frac{c_{\rm p}\rho^2\kappa z^2}{12acT^3}\left(\frac{g\delta}{2H_{\rm p}}\right)^{1/2}. \quad (5)$$

An inportant characteristic of the CGM model is the absence of free parameters. In particular, the mixing length is taken as $l=z$ where z is the geometrical distance from the top of the convection zone. Stothers & Chin (1997) have recently shown that $l=z$ performs very well in different kind of stars. In order to avoid numerical difficulties, we allowed for a very small overshooting at the moment of evaluating l, that is, we considered $l=z+\beta H_{\rm p}^{\rm top}$, where $H_{\rm p}^{\rm top}$ is the pressure scaleheight at the top of the outer convection zone. We chose $\beta<0.1$, which does not affect the temperature profile of the convection zone of our evolving models.

For the sake of comparison, we also included in our calculations the ML1, ML2 and ML3 parametrizations of MLT. These versions, which are associated with different convective efficiencies, has been amply employed in white dwarf investigations (see Tassoul et al. 1990 for details).

3 EVOLUTIONARY RESULTS

Here, we present the main results of our calculations. We computed the evolution of DA white dwarf models with masses ranging from $M=0.5$ to 1.0 $M_{\odot}$ at intervals of 0.1 $M_{\odot}$ and with a metallicity of $Z=0.001$. We varied $M_{\rm H}$ within the range $10^{-13}\le M_{\rm H}/M\le 2\times 10^{-4}$ and $M_{\rm He}$ in the range $10^{-6}\le M_{\rm He}/M\le 10^{-2}$. We employ the FST approach given by CGM, and also the ML1, ML2 and ML3 versions of MLT. The models were evolved from the hot white dwarf stage down to $\log (L/L_{\odot})=-5$. Unless stated otherwise, we shall refer in what follows to those models parametrized by $M_{\rm H}=10^{-6}M$ and $M_{\rm He}=10^{-2}M$.

We begin by examining Fig. 1 in which the surface gravity of our DA models with $\log q({\rm H})=-10$, -6, -4 and no hydrogen envelope is depicted in terms of the effective temperature (q is the external mass fraction given by $q=1-M_r/M$).[1] The first observation we can make from this figure is that thick hydrogen envelopes appreciably modify the surface gravity of no-hydrogen models, especially in the case of low-mass configurations. The effect of adding an outer hydrogen envelope of $M_{\rm H}=10^{-10}M$ is barely noticeable. Note also that finite-temperature effects are relevant even in massive models (see also Koester & Schönberner 1986). At low effective temperatures, convective mixing between hydrogen and helium layers (see later in this section) changes the surface gravity of models with thin hydrogen envelopes (see the discussion in Benvenuto & Althaus 1998 in the context of low-mass helium white dwarfs for more details about this topic).

It is worthwhile to mention that our more massive models begin to develop a crystalline core at effective temperatures close to the temperature range of the ZZ Ceti instability strip. More specifically, the onset of crystallization at the centre of 1.0-, 0.9-, 0.80- and 0.7-$M_{\odot}$ DA models with $\log q=-6$ takes place at $T_{\rm eff}=16\,200$, 13 100, 10 870 and 8900 K, respectively. For details concerning the process of growth of the crystal phase of our models, see Benvenuto & Althaus (1995).

The role played by hydrogen burning in our evolving models is worthy of comment. Iben & Tutukov (1984) were the first in drawing attention to the fact that hydrogen burning in cooling white dwarfs could be, at low luminosities, a

[1]For the sake of clarity, in this and the following figures we do not depict the results corresponding to 1.0-$M_{\odot}$ models.

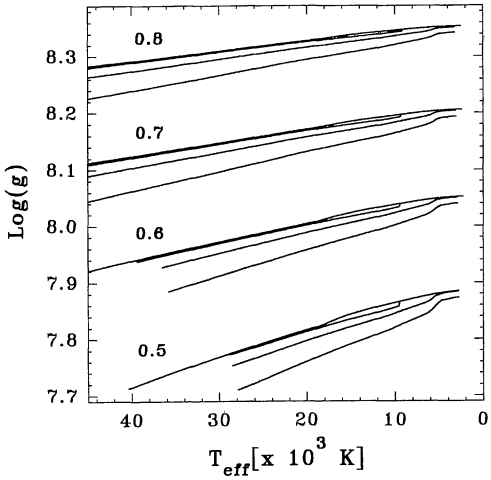

Figure 1. Surface gravity versus effective temperature for DA white dwarf models with (from top to bottom) $M/M_{\odot}=0.8$, 0.7, 0.6 and 0.5. For each stellar mass, the four curves correspond to models with (from top to bottom) no hydrogen envelope, and $\log q({\rm H})=-10$, -6 and -4. Note that very thick hydrogen envelopes appreciably modify the surface gravity of the no-hydrogen models.

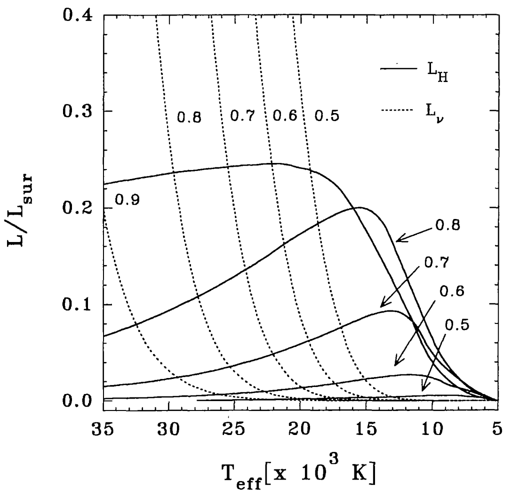

Figure 2. Hydrogen-burning (solid lines) and neutrino luminosity (dotted lines), (in terms of the surface photon luminosity) as a function of the effective temperature for 0.5-, 0.6-, 0.7-, 0.8- and 0.9-$M_{\odot}$ DA white dwarf models with $\log q({\rm H})=-4$.

major energy source. We find, in particular, that for models with $\log q({\rm H})\gtrsim -4$, hydrogen burning may contribute non-negligibly to the surface luminosity. For more detail, we depict in Fig. 2 the relative contribution of hydrogen burning as a function of effective temperature for various stellar masses having a hydrogen envelope of $\log q({\rm H})=-4$. Note that for intermediate stellar masses the hydrogen-burning contribution reaches a maximum near the ZZ Ceti instability strip. Another feature worthy of mention shown by Fig. 2 is that, even for more massive models, hydrogen burning never becomes the dominant source of stellar energy. We also include in Fig. 2 the neutrino luminosity of the models. For the effective temperature range shown in the figure, neutrino luminosity strongly decreases with cooling and it is negligible by the time models enter the instability strip.

The importance of nuclear burning is strongly sensitive to the exact value of the hydrogen layer mass, as was recognized by Koester & Schönberner (1986) (see also D'Antona & Mazzitelli 1979). To show this, we have computed additional sequences by considering the cases with $\log q(\mathrm{H}) = -3.824$ and -3.699. The results for 0.6- and 0.8-$M_\odot$ models are illustrated in Fig. 3. It is clear that a small difference in the hydrogen layer mass is responsible for the different role of hydrogen burning. In particular, for the 0.6-$M_\odot$ model with $M_\mathrm{H} = 9 \times 10^{-5}$ $M_\odot$, we find that the relative contribution of hydrogen burning at low luminosities remains always below 9 per cent, while for the same model but with $M_\mathrm{H} = 1.2 \times 10^{-4}$ $M_\odot$, the hydrogen-burning contribution rises up to ≈ 18 per cent.

The behaviour of the evolving outer convection zone is depicted in Figs 4–7 for DA white dwarf models with $M = 0.5$, 0.6, 0.8 and 1.0 $M_\odot$ and for the different theories of convection. In each figure, we plot the location of the top and the base of the outer convection zone in terms of the mass fraction q as a function of effective temperature. The first observation we can make from these figures is that at both hot and cool extremes the convection zone profile is independent of the treatment of convection. In fact, at high effective temperatures only a negligible fraction of energy is carried by convection and, consequently, the temperature stratification remains essentially radiative, while at low temperatures convection is very efficient and most of the convection zone assumes an adiabatic stratification. At intermediate effective temperatures, however, where the ZZ Ceti stars are observed, convection treatment is decisive in fixing the structure of the outer zone of the models. Note that, in contrast to the situation encountered for DB objects (Benvenuto & Althaus 1997), the CGM predictions are roughly intermediate to those of the ML1 and ML2 convection. Another feature shown by these figures and worthy of comment is that the thickness of the convection zone in the CGM model begins to increase at a given effective temperature much more steeply than in any of the MLT versions. A similar trend is also found in DB white dwarf models, though it is worth remarking that the global behaviour of the evolving convection zone is markedly different in both types of star. In agreement with previous results (D'Antona & Mazzitelli 1979), the final extent reached by the base of the convection zone is smaller in the more massive models. We should mention that the small jump at the top of the convection zone at $T_\mathrm{eff} \approx 9000$ K is brought about, not surprisingly, by the change from OPAL to Alexander & Ferguson's (1994) molecular opacities. This jump affects the z values, thus giving rise to some irregularities in the CGM convection profile, which does not alter the conclusions of this work.

To clarify better the role played by convective efficiency in determining the thermal structure of the outer layers, we

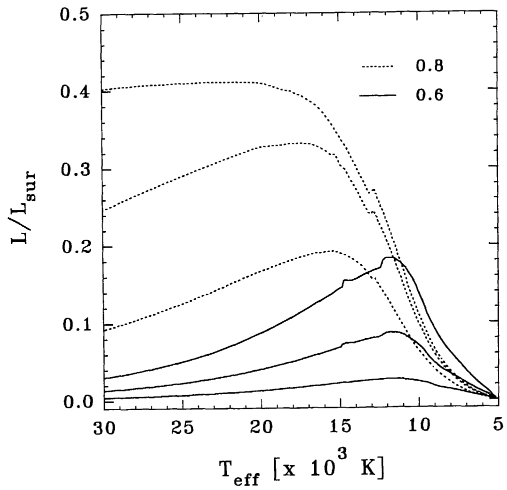

Figure 3. Hydrogen-burning luminosity (in terms of the surface photon luminosity) as a function of the effective temperature for 0.6- and 0.8-$M_\odot$ DA white dwarf models. For each stellar mass and from top to bottom, models with $\log q(\mathrm{H}) = -3.699$, -3.824 and -4 are depicted.

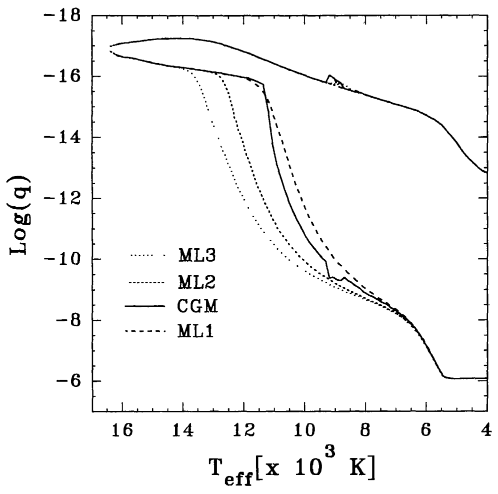

Figure 4. The location of the top and the base of the outer convection zone expressed in terms of the mass fraction q versus the effective temperature for a 0.6-$M_\odot$ DA white dwarf model according to different theories of convection. For the small jump at $T_\mathrm{eff} \approx 9000$ K, see text.

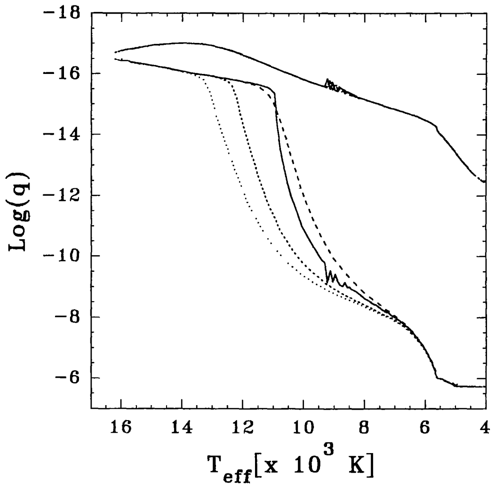

Figure 5. Same as Fig. 4, but for a 0.5-$M_\odot$ DA white dwarf model.

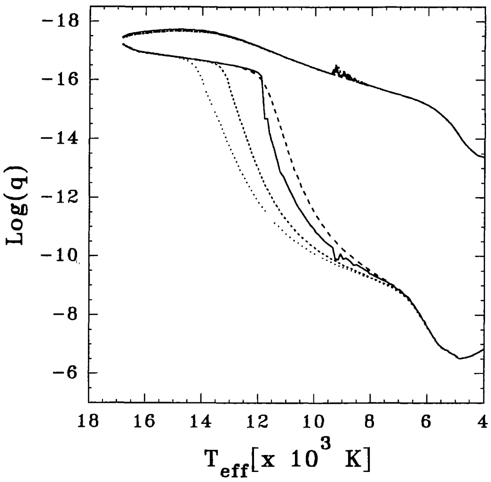

Figure 6. Same as Fig. 4, but for a 0.8-$M_{\odot}$ DA white dwarf model.

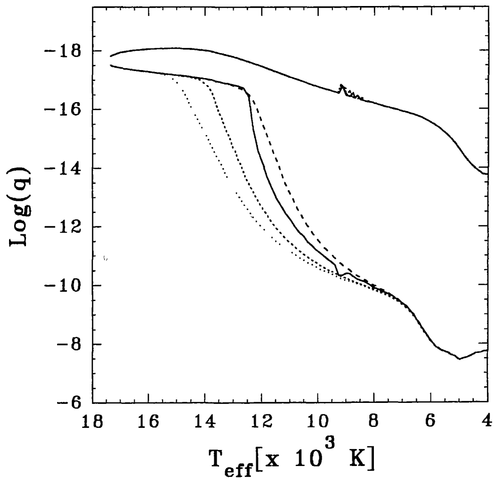

Figure 7. Same as Fig. 4, but for a 1.0-$M_{\odot}$ DA white dwarf model.

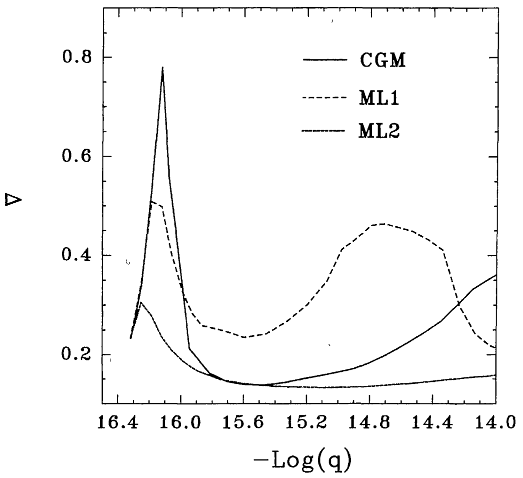

Figure 8. The convective temperature gradient as a function of the mass fraction q for the 0.6-$M_{\odot}$ DA white dwarf model at $T_{eff}=10\,890$ K according to CGM and the ML1 and ML2 versions of MLT. The fact that $l<H_p$ in the outermost layers leads to a strong peak in ∇ in the case of the CGM model.

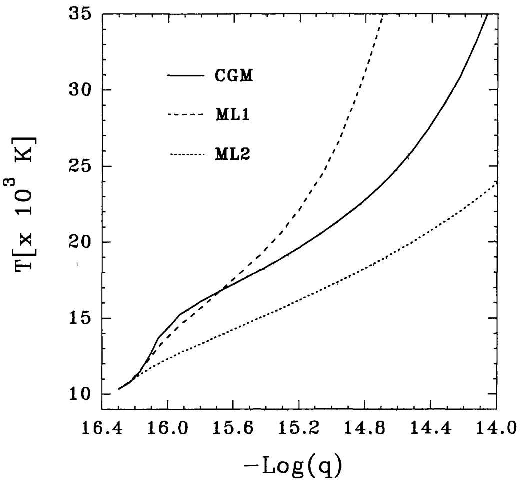

Figure 9. The outer layer temperature versus the mass fraction q. The DA model corresponds to that of Fig. 8. Note that the temperature profile given by CGM cannot be reproduced by MLT with a single value of α.

show first of all in Fig. 8 the dimensionless convective temperature gradient in terms of the mass fraction q for the outer layers of a 0.6-$M_{\odot}$ DA model with $T_{eff}=10\,890$ K according to the CGM model and the ML1 and ML2 versions of MLT. Note the presence of a strong peak in ∇ at $\log q \approx -16.1$ in the CGM case. This arises simply because $z<H_p$ in the outermost layers, thus giving rise to very inefficient convection. In somewhat deeper layers, however, the CGM model turns out to be more efficient than ML1. This is a result of the fact that the larger number of eddies characterizing the CGM model, compared with MLT, begins to play a significant role. For $\log q > -15.6$, both the CGM and ML2 models provide an adiabatic stratification and hence the behaviour of ∇ in the outermost layers will be critical for determining the temperature profile in the envelope and thus the extent of the convection zone of the model. The resulting temperature profile of the model analysed in Fig. 8 is depicted in Fig. 9. It is clear that the CGM model predictions are very different from those of the ML1 and ML2 models. Indeed, the temperature profile cannot be reproduced with an MLT model with a single value of α.

A final remark about the convection profiles is that at low effective temperatures they ultimately reach a maximum depth, which is determined by the location of the boundary of the degenerate core. For the 0.60-$M_{\odot}$ DA white dwarf model, in particular, such a depth occurs around $\log q \approx -6$ (see Tassoul et al. 1990 for a similar result). Accordingly, mixing episodes between hydrogen-rich and the underlying helium-rich zones are possible only for values of $\log q$(H) lower than this limit. It is apparent from Figs 4–7 that the mixing effective temperature for models with very thin hydrogen envelopes will be strongly dependent on the assumed convective efficiency and not so much on the stellar mass. To put this assertion on a more quantitative basis,

Table 1. Mixing temperatures and final hydrogen abundance for different models and convection theories.

Sequence	ML1	ML2	ML3	CGM
0.50 (-12)	10,000 (<1e-6)	11,240 (<1e-6)	11,870 (<1e-6)	10,400 (<1e-6)
0.50 (-10)	9130 (1e-5)	10,140 (9e-6)	10,700 (9e-6)	9500 (1e-5)
0.50 (-8)	7090 (2e-3)	7450 (1.7e-3)	7470 (1.7e-3)	7220 (1.7e-3)
0.50 (-6)	5600 (≈0.55)	5605 (≈0.55)	5610 (≈0.55)	5610 (≈0.55)
0.60 (-13)	10,520 (<1e-6)	11,820 (<1e-6)	12,505 (<1e-6)	10,910 (<1e-6)
0.60 (-12)	10,130 (<1e-6)	11,380 (<1e-6)	12,000 (<1e-6)	10,570 (<1e-6)
0.60 (-11)	9680 (3e-6)	10,840 (2e-6)	11,450 (2e-6)	10,020 (3e-6)
0.60 (-10)	8950 (3e-5)	10,090 (2e-5)	10,650 (2e-5)	9450 (2e-5)
0.60 (-8)	6710 (7.3e-3)	6900 (6.4e-3)	6850 (6.8e-3)	6780 (6.6e-3)
0.60 (-6)	5330 (≈0.8)	5360 (≈0.8)	5380 (≈0.8)	5360 (≈0.8)
0.70 (-12)	10,160 (1e-6)	11,390 (1e-6)	12,100 (1e-6)	10,640 (1e-6)
0.70 (-10)	8980 (8e-5)	9890 (6e-5)	10,330 (6e-5)	9320 (7e-5)
0.70 (-8)	6410 (0.027)	6530 (0.025)	6530 (0.026)	6500 (0.025)
0.80 (-12)	10,250 (2e-6)	11,500 (1e-6)	12,020 (1e-6)	10,760 (1e-6)
0.80 (-10)	8740 (2.2e-4)	9740 (1.7e-4)	10,010 (1.6e-4)	9180 (1.8e-4)
0.80 (-8)	6320 (0.1)	6330 (0.1)	6350 (0.1)	6350 (0.1)
0.90 (-12)	10,350 (4e-6)	11,470 (3e-6)	12,050 (3e-6)	10,830 (3e-6)
0.90 (-10)	8560 (6.5e-4)	9320 (5e-4)	9490 (4.6e-4)	9120 (5e-4)
0.90 (-8)	6170 (≈0.4)	6200 (≈0.4)	6220 (≈0.4)	6230 (≈0.4)
1.0 (-12)	10,290 (1e-5)	11,560 (7e-6)	11,980 (7e-6)	10,800 (8e-6)
1.0 (-10)	8240 (2e-3)	8810 (1.6e-3)	8950 (1.6e-3)	8610 (1.6e-3)
1.0 (-8)	5950 (≈0.8)	5970 (≈0.8)	5990 (≈0.8)	6000 (≈0.8)

Note. Each sequence is denoted by the stellar mass (in solar mass units) and the value of $\log q_{\rm H}$. The numbers in parentheses next to each temperature give the hydrogen surface content after mixing.

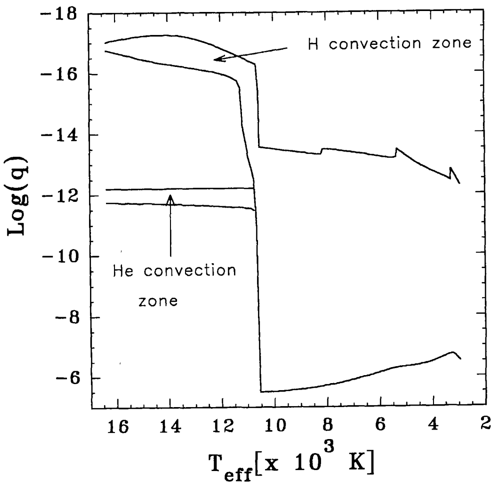

Figure 10. The location of the top and the base of the outer convection zones expressed in terms of the mass fraction q versus the effective temperature. The results correspond to a 0.6-$M_\odot$ DA white dwarf model with $\log q(\rm H) = -12$ according to the CGM model. Note the presence of two convection zones at high $T_{\rm eff}$ which ultimately merge at $T \approx 10\,500$ K, thus giving rise to a drastic change of the surface chemical composition from a hydrogen-dominated to a helium-dominated one.

we list in Table 1 the effective temperatures at which the hydrogen surface abundance begins to decrease appreciably as a result of convective mixing, together with the final hydrogen abundances (for models with thick hydrogen layers, the mixing process takes place gradually in a finite range of effective temperatures; in that case we only give approximate values of the hydrogen abundance). Clearly, for models with very thin hydrogen envelopes convective mixing drastically modifies the surface composition from a hydrogen-dominated to a helium-dominated one. This behaviour can be understood directly by examining Fig. 10, which shows the evolving convection zones of the 0.6-$M_\odot$ model with $\log q(\rm H) = -12$ according to CGM convection. At $T_{\rm eff} \approx 10\,600$ K, the merging of the hydrogen convection zone with the helium convection zone (located just below the hydrogen/helium transition region) causes the base of the convection zone to reach very deep helium layers quickly, giving rise to an almost complete dilution of the hydrogen content. From then on, the subsequent evolution corresponds to that of a DB model (see fig. 11 of Benvenuto & Althaus 1997).

In order to estimate approximate effective temperatures for the theoretical blue edge of the DA instability strip, we employ thermal time-scale arguments $\tau_{\rm th}$ for our evolving models. Several past studies (Cox 1980; Winget et al. 1982; Tassoul et al. 1990 and references cited therein) have shown that the behaviour of pulsational instabilities in white dwarf stars can be understood in terms of the $\tau_{\rm th}$ of the driving regions, defined by

$$\tau_{\rm th} = \int_0^{q_{\rm bc}} \frac{C_{\rm V} T}{L} M_* dq. \qquad (6)$$

In equation (6), $C_{\rm V}$ is the specific heat at constant volume, L

Table 2. Theoretical blue-edge effective temperatures versus stellar mass and convection theory for our DA white dwarf

Theory of convection	Mass ($M/M_\odot$)	$T_{\rm eff}$ (K)
CGM	0.50	10,710
ML1	"	10,400
ML2	"	11,630
ML3	"	12,310
CGM	0.60	10,970
ML1	"	10,600
ML2	"	11,850
ML3	"	12,530
CGM	0.70	11,210
ML1	"	10,800
ML2	"	12,050
ML3	"	12,710
CGM	0.80	11,430
ML1	"	10,980
ML2	"	12,240
ML3	"	12,890
CGM	0.90	11,650
ML1	"	11,160
ML2	"	12,430
ML3	"	13,070
CGM	1.0	11,870
ML1	"	11,350
ML2	"	12,620
ML3	"	13,250

and M are, respectively, the luminosity and the mass of the model, and the subscript 'bc' corresponds to the location of the bottom of the superficial convection zone. In particular, the blue edge of the instability strip (the onset of pulsations) corresponds approximately to the effective temperature at which $\tau_{\rm th}$ becomes comparable to 100 s, which for a white dwarf is of the order of the shortest observable g-mode periods. In this connection, we list in Table 2 (see also Fig. 11) the effective temperature of our theoretical blue edges of the ZZ Ceti instability strip acording to the different stellar masses and convection treatments we have considered. Note, as found by other investigators (e.g. Tassoul et al. 1990 and Bradley & Winget 1994) the dependence of the blue edge temperature on the stellar mass and, more importantly, on the assumed convective efficiency. In particular, our MLT blue edges are consistent with those of Bradley & Winget (1994) obtained on the basis of detailed pulsation calculations. Clearly the CGM theory predictions are intermediate to the ML1 and ML2 results. In Fig. 11, we have also included the theoretical result obtained by Gautschy et al. (1996) for a 0.6-$M_\odot$ ZZ Ceti model and the observational data of Bergeron et al. (1995) as well (we picked out those ZZ Ceti stars analysed by Bergeron et al. which, for a given stellar mass, are characterized by the highest temperature).

With regard to the inclusion of the observational data in this figure, some comments seem to be appropriate at this point. As mentioned earlier, the effective temperature of the stars that define the *observational* blue edge is computed, as usual, by employing the emergent spectrum of a model atmosphere, which in the case of DA white dwarfs is

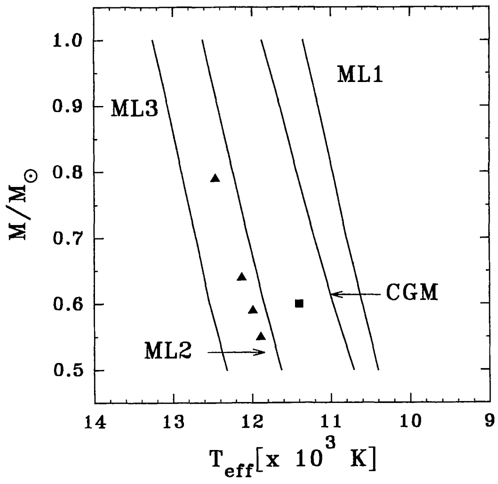

Figure 11. The dependence of theoretical blue edge temperature on stellar mass for CGM, and for the ML1, ML2 and ML3 versions of MLT. The filled triangles represent the observations of Bergeron et al. (1995) for (from top to bottom) G226-29, G185-32, R548 and G238-53. The filled square corresponds to the blue edge temperature obtained by Gautschy et al. (1996) from hydrodynamical simulations of convection. Note that both the CGM model and the hydrodynamical simulation tend to point towards a cooler blue edge than the observed one.

strongly dependent on the prescription adopted for the treatment of convection (see e.g. Bergeron et al. 1995). Thus, in order to perform a self-consistent comparison with observations, we should employ a model atmosphere computed considering the CGM theory. In view of the fact that such models are still not available, any comparison of our theoretical results with observation should be taken with caution.

In spite of the above warnings, it is remarkable that both the CGM theory and Gautschy et al.'s (1996) predictions tend to point towards a cooler blue edge than observations. In particular, on the basis of very detailed hydrodynamical simulations of convection in a 0.6-$M_\odot$ ZZ Ceti model, Gautschy et al. derived an effective temperature between 11 400 and 11 800 K for the blue edge. As far as the blue edge dependence on the stellar mass is concerned, the MLT and CGM theory predictions are rather similar, spanning $\approx$1000 K over the range of stellar masses that we considered. Note that the dependence of the blue edge on the stellar mass predicted by both theories of convection is rather similar to that shown by observations.

4 SUMMARY

In this paper we compute the structure and evolution of carbon–oxygen white dwarf models with hydrogen envelopes (DA type). We consider stellar masses ranging from $M=0.5$ to 1.0 $M_\odot$ at intervals of $M=0.1$ $M_\odot$, and we treat the masses of the hydrogen and helium envelopes as free parameters within the range $10^{-13} \le M_{\rm H}/M \le 10^{-4}$ and $10^{-6} \le M_{\rm He}/M \le 10^{-2}$, respectively. The models were evolved from the intermediate effective temperature stage down to $\log(L/L_\odot) = -5$.

The calculations were made with a white dwarf evolutionary code including updated radiative and conductive opacities and neutrino emission rates, and very detailed equations of state for hydrogen and helium plasmas. We also include the effects of crystallization, convective mixing and hydrogen burning (pp chain and CNO bi-cycle). The most important feature of our study, however, is that we treat the energy transport by convection within the formalism of the so-called full-spectrum turbulence theory, which constitutes an improvement over most previous studies of DA white dwarf evolution. In particular, we employ a new model based on such theory developed by CGM. This new model includes the full spectrum of eddies, has no free parameter and computes the rate of energy input self-consistently. In the white dwarf domain, the CGM model has been recently shown (Althaus & Benvenuto 1997b) to provide a good fit to new observational data of pulsating DB objects. Finally, for the sake of comparison, we also consider the most common parametrizations of mixing length theory (MLT) usually employed in this kind of study.

In agreement with previous studies, we find that very thick hydrogen layers substantially modify the surface gravity of the models and that the importance of nuclear burning is strongly sensitive to the exact value of the hydrogen layer mass. In particular, we find that for the 0.6-$M_\odot$ model with $M_H = 9 \times 10^{-5}\,M_\odot$ the relative contribution of hydrogen burning at low luminosities always remains below 9 per cent, while for the same model but with $M_H = 1.2 \times 10^{-4}\,M_\odot$, the hydrogen-burning contribution rises up to ≈ 18 per cent.

One of our main interests in this work has been to study the evolution of ZZ Ceti models with the aim of comparing the CGM and MLT predictions. In this connection, we find that the temperature profile given by the CGM model is markedly different from that of the ML1 and ML2 models, and it cannot be reproduced by any MLT model with a single value of α. The evolving outer convection zone also behaves differently in both theories. In particular, the thickness of the convection zone in the CGM model begins to increase at a given effective temperature much more steeply than in any of the MLT versions and remains intermediate to those with ML1 and ML2 convection.

We have also computed approximate effective temperatures for the theoretical blue edge of the DA instability strip by using thermal time-scale arguments for our evolving DA models. In this context, we find that the effective temperature of the theoretical blue edge of the ZZ Ceti instability strip depends strongly on the stellar mass and more importantly on the assumed convective efficiency, in agreement with previous studies. In particular, our MLT blue edges are consistent with those of Bradley & Winget (1994) obtained on the basis of detailed pulsation calculations. We find that CGM theory predicts blue edges cooler (by ≈ 1000 K) than the observed ones. Specifically, we find an effective temperature of 11 000 K for the blue edge of a 0.6-$M_\odot$ DA model. It is remarkable that recent non-adiabatic pulsation calculations based on numerical simulations of convection tend also to indicate a somewhat cooler blue edge. However, we remind the reader that the physical ingredients we employed in the present paper are *not* consistent with those of the stellar atmosphere calculations employed by Bergeron et al. (1995), particularly in the treatment of convection. Although a difference of 1000 K between the effective temperature of the observed and theoretical blue edges for the ZZ Ceti instability strip seems to be large, we should wait for model atmospheres also computed in the frame of the full-spectrum turbulence theory. Only after such models become available shall we be able to gauge the actual importance of such a discrepancy.

Detailed tabulations of the evolutions of our DA models, which are not reproduced here, are available upon request from the authors at their e-mail address.

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